Paris, Septembre 5, 1999

Dear Richard,

Thank you for your work on the Torino scale: such an scale was clearly missing. May I contribute with a small comment?

The horizontal scale of the plot is logarithmic, as, in this way, small probabilities are conveniently represented. Unfortunately, the logarithmic scale treats unfairly 'large' probabilites, i.e., probabilities close to one. You circumvent this difficulty in the plot by introducing a zone $P > 0.99$.

In fact, I see this as a classical difficulty, that can be solved using an alternative definition of probability. The definition can be formalized (in the Kolmogorov sense), but, crudely speaking, instead of defining a *probability* in the usual manner,

$$\begin{aligned} P &= \frac{\text{number of positive cases}}{\text{total number of cases}} \\ &= \frac{\text{number of positive cases}}{\text{number of positive cases} + \text{number of negative cases}} \ , \end{aligned} \tag{1}$$

one may define the *eigenprobability* (Tarantola, 1999) as

$$\chi = \frac{\text{number of positive cases}}{\text{number of negative cases}} \ . \tag{2}$$

The domains where P and χ take values is

$$0 \leq P \leq 1 \qquad ; \qquad 0 \leq \chi \leq \infty \quad , \tag{3}$$

and the two definitions of probability are related through

$$\chi = \frac{P}{1-P} \qquad ; \qquad P = \frac{\chi}{1+\chi} \quad . \tag{4}$$

While the logarithmic probability $\widehat{P} = \log P$ is not an interesting definition, the *logarithmic eigenprobability*, defined as

$$\widehat{\chi} = \log \chi \ , \tag{5}$$

has a 'nice' behaviour both for low eigenprobabilities $(\chi \to 0)$ and large eigenprobabilities $(\chi \to \infty)$.

The following table shows the relative bahaviour of the two variables $\widehat{\chi}$ and P (I use here base 10 logarithms, familiar to engineers):

$\widehat{\chi} = \log \chi$	P
-4	0.0001
-3	0.001
-2	0.01
-1	0.1
0	0.5
1	0.9
2	0.99
3	0.999
4	0.9999

.

You see that when the values of $\widehat{\chi}$ increase indefinitely, the probability P indefinitely approaches one.

The figure here below proposes a slight modification of the Torino scale, where the low and the high probabilities are treated equivalently. If, some unfortunate day, we must evaluate 'large' probabilities of impact $(P \to 1)$, $P = 0.9$ is quite different from $P = 0.99$, that is itself quite different from 0.999 . These values deserve a right placing on the scale.

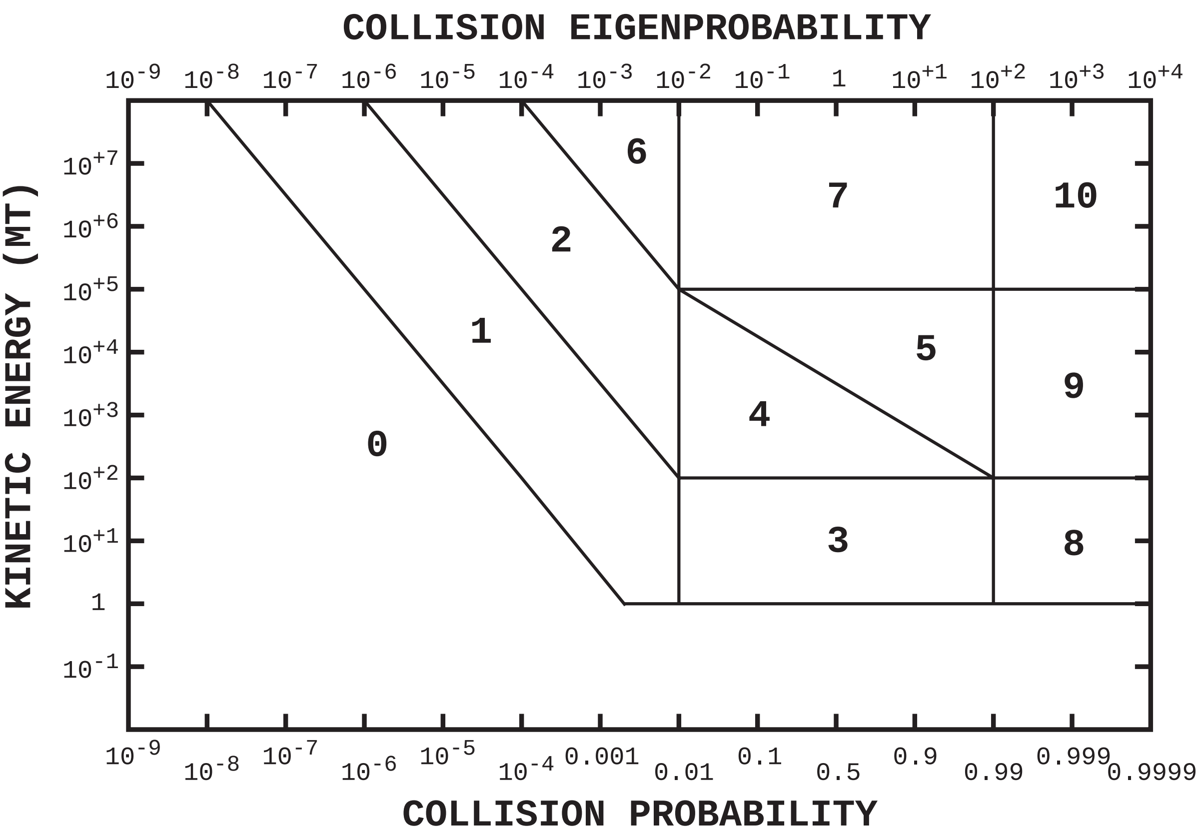

Figure 1: A slight modification of the Torino scale (Richard Binzel). The axis at the bottom is tabulated in terms of the usual probability P , while the axis at the top corresponds to the eigenprobability $\chi = P/(1-P)$. The scale is linear in $\log \chi$.