Using Pulsars to Define Space-Time Coordinates

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Abstract

Fully relativistic coordinates have been proposed for (relativistically) running a "GPS" system. These coordinates are the arrival times of the light signals emitted by four "satellites" (clocks). Replacing the signals emitted by four controlled clocks by the signals emitted by four pulsars defines a coordinate system with lower accuracy, but valid across the whole Solar System. We here precisely define this new coordinate system, by choosing four particular pulsars and a particular event as the origin of the coordinates.

We are all familiar with the three coordinates used to represent the position of points in the laboratory, on the Earth, or in the sky. Relativity theory has taught us that time is not to be viewed as flowing independently of space, and physicists routinely use the notion of event: a point in the four-dimensional continuum introduced by Poincaré (later called space-time by Minkowski), where an event has four coordinates, say (x, y, z, t). An experimental protocol allowing to attach such four coordinates to an event, using clocks and light signals, was exposed by Einstein.

One of the major technical advances in the recent years has been the development of Global Navigation Satellite Systems: GPS in the USA, Glonass in Russia, and Galileo (under construction) in the EU. These systems provide the coordinates of any point on the Earth, as well as an 'universal time'. Unfortunately, these coordinates do not qualify as fully relativistic: in Minkowski space-time, durations and distances depend on the observer, and are not universal. While the Minkowski coordinates (x, y, z, t) of an event are bona-fide relativistic coordinates, they are not i

Is it possible to define in the space-time an immediate system of four relativistic coordinates? The answer is yes. Four clocks in arbitrary motion in space-time, broadcasting their proper time constitute such a system: the coordinates of a point in space-time (i.e., of a space-time event) are, by definition, the four times $(\tau_1, \tau_2, \tau_3, \tau_4)$ of the four signals converging at that space-time point, as recorded by a standard receiver. So any observer able to receive the signals is able to (instantaneously) know its own coordinates, in fact, its own space-time trajectory, expressed in these 'light-coordinates'[1].

But how could these coordinates be used to do space-time geometry, i.e., how could them be related to the observer's local unit of time and of distance? We have suggested (Coll, 2000; Coll et al., 2006, Tarantola et al. 2009), to take this point of view as the basic paradigm for 'running' a Global Positioning System. Imagine that four 'satellites' wander in space-time, emitting their proper times. If, in addition to broadcasting its own time signal, each satellite sends an echo of the signals received from the other three, then, any observer able to receive the signals of the four satellites knows their trajectory in this self-consistent coordinate system. Then, this observer is able to write the components of the space-time metric tensor at the space-time point where the observer is. In other words, the observer can use the light-coordinates to build a local clock and, therefore, a local meter[2].

The agency in charge of running a satellite positioning constellation should only care on the quality of the embarked clocks, and the quality of the broadcasted signals. Relating these light-coordinates to any terrestrial coordinate system (either a global geographical system or a local one) is just an attachment problem, that should not interfere with the problem of defining the primary coordinate system itself. Under some simple assumptions about the gravity field around the Earth, four satellites define a usable coordinate system. Redundancies created by supplementary satellites should allow to model the gravity field itself. Some day, all positioning satellite systems will be run this way.

Besides this, can we today build a relativistic immediate coordinate system valid in a large domain (larger than our Solar system)? Yes, if instead of using the signals coming from manufactured and controlled clocks we use the natural objects that best approach this: pulsars.

[1]We say 'light-coordinates because one would typically use electromagnetic waves to propagate the signals.

[2]This, of course, is only exact if the gravitational field is given. In Tarantola et al. (2009) a more general setting is proposed, where the space-time metric itself is evaluated. For other details, see http://www.coll.cc.

Pulsars are rotating astronomical objects, located in the Galaxy, that emit quasi-periodic signals. Of particular interest to us are the millisecond pulsars (with a period of the order of 1 ms). The main source of variability in the pulsars' signals is the interstellar medium, that imposes an uncertainty in the measurement of arrival times of the order of 4 nanoseconds.

We propose to define a Universal (well, say Galactic) system of space-time coordinates as follows. By convention we select the four millisecond pulsars 0751+1807 (3.5 ms), 2322+2057 (4.8 ms), 0711-6830 (5.5 ms) and 1518+0205B (7.9 ms). Their angular distribution around the Solar system is quite even (they look almost like the vertices of a tetrahedron seen from its center). We define the origin $(\tau_1, \tau_2, \tau_3, \tau_4) = (0, 0, 0, 0)$ of the space-time coordinates as the event *0H0'0", January 1, 2001*, at the focal point of the Cambridge radiotelescope (the one that was used for the discovery of the pulsars). Then, any other space-time event, on Earth, on the Moon, anywhere in the Solar system or in the solar systems in this part of the Galaxy, has its own coordinates attributed. With present-day technology, this locates any event with an accuracy of the order of 4 ns, i.e., of the order of one meter. This is not an extremely precise coordinate system, but it is extremely stable and has a great domain of validity.

Imagine when spacecrafts that —like Pioneer Voyager— leave our solar system, are equipped with a receiver of the signals from the four reference pulsars: in their messages to the Earth they will then include their own space-time coordinates.

If, tomorrow, a human colony is installed in, say, Pluto, with radiotelescopes able to receive the signals of the four reference pulsars. It would not be sufficient to e-mail this *arXiv* posting to the Pluto colony in order for them to be able to use these coordinates: one vessel must have travelled from the Earth to Pluto, continuously recording the pulsar signals, so the origin of the coordinates would have been 'transported' to Pluto. Only then, the Pluto colony would share a common coordinate system with the mainland.

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References

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