COURSE 7

OCEAN SEISMO-ACOUSTIC MODELING:
NUMERICAL METHODS

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1. Introduction

It is well established that all types of electromagnetic waves (light included) propagate poorly in turbid, saline sea water, and that only acoustic waves constitute an effective tool for communicating and sensing at long range underwater [1–4]. The use of sound to explore the ocean and the sea floor is an ever-evolving science, which has seen remarkable progress in military as well as in civilian applications over the past few decades. Since World War II, naval interests in developing still better and more reliable sonar systems for submarine detection have provided most of the impetus (and funding) for the basic research in ocean acoustics. To achieve optimum sonar design one needs to know the frequency-dependent propagation characteristics over long distances in the ocean for a variety of different environmental conditions. In the civilian sector, underwater-sound devices have found application in depth sounding, sea-floor mapping, subbottom profiling, fish finding, etc.

The range of useful frequencies in underwater acoustics covers almost five decades of frequency — from a few hertz to several tens of kilohertz — with wavelengths from around 1 km down to a few centimeters. Since attenuation of sound in sea water increases approximately with frequency squared, the high-frequency devices (> 10 kHz) are used primarily in short-range high-resolution systems such as bottom profilers and fish sonars. The lower frequencies propagate to long ranges with little attenuation and are therefore preferred in passive surveillance systems for military use. It is the long-range propagation problems at low and intermediate frequencies (< 1 kHz) that shall be addressed in this paper.

The ocean is an inhomogeneous, irregular waveguide limited above by a reflecting boundary (the sea surface) and below by a penetrable boundary (the sea floor). The forward modeling problem consists in solving the acoustic wave equation with appropriate boundary and radiation conditions in order to estimate the frequency-dependent spatial and temporal properties of the sound pressure field in the ocean waveguide. This problem is generally too complex for analytical solutions, and hence we must resort to numerical methods.

The development of a seismo-acoustic modeling capability in ocean
acoustics has been in continuous expansion over the past 20 years, trailing closely the stunning advances in computer technology. Up to around 1970 the only practical technique for solving propagation problems in underwater acoustics was based on ray theory, which is a computationally efficient but approximate (infinite frequency) solution of the wave equation. The ray techniques are still in use for solving high-frequency deep-water problems in ocean acoustics.

In the early 1970s powerful digital computers became available in most research establishments, stimulating the development of more accurate frequency-dependent solutions of the wave equation. These wave-theory solutions (all numerically based) encompass normal-mode, fast field, and parabolic equation techniques [5]. Today general-purpose numerical codes based on the above solution techniques are widely used for modeling low-frequency propagation in complex ocean environments.

Even though wave-theory models provide accurate field solutions at any source frequency, the required computational effort is strongly frequency dependent. At best the calculation time increases linearly with frequency. However for most solution techniques the calculation time increases with frequency squared, which in practice restricts the applicability of these models to low-frequency problems. In any event, with present-day computing power it is entirely feasible to apply wave-theory models to deep-water problems (H ~ 5000 m) for frequencies up to a few hundred hertz, and to shallow-water problems for frequencies up to a few kilohertz.

The structure of this paper is the following: In section 2 we briefly describe important propagation and loss mechanisms in the ocean. Section 3 presents an overview of computer-implemented solutions of the time-independent wave equation. The range-independent models are discussed in detail in section 4 and the range-dependent models in section 5. Next we address the question of pulse modeling in ocean acoustics (section 6) based on Fourier synthesis of time-harmonic solutions. Section 7 is dedicated to two numerical examples illustrating how the geoaoustic parameters of the seabed can be determined from broadband acoustic or seismic data through iterative forward modeling. The paper ends with a summary and conclusions.

2. The ocean waveguide

The goal of ocean acoustic modeling is to estimate the frequency dependent spatial and temporal properties of the sound pressure fields in ocean waveguides. To illustrate the complexity of this modeling problem, let us first look at the environmental complexity of real ocean waveguides.

The ocean itself is an environment with both temporal and spatial variability not always suited for a deterministic modeling approach. As an example let us look at some thermistor chain data recorded recently in the Norwegian Sea [6]. Figure 1 displays three different temperature-profile sections each covering a range of approximately 50 km and depths between 15 and 150 m. Even though these data were all recorded in the same area within a period of two weeks in June 1987, we notice strong differences in lateral variability along the three tracks. Thus Fig. 1(a) displays a stable water column well suited for deterministic modeling at all frequencies of practical importance in ocean acoustics. Figure 1(b), on the other hand, shows a section with a stable surface layer, but with some microstruc-
ture variability below the thermocline causing scattering of high-frequency sound. Hence this situation is suited for deterministic modeling only at intermediate and low frequencies. Finally, fig. 1(c) shows a section with strong lateral variability suited only for a stochastic modeling approach, except, maybe, at very low frequencies.

An important issue is that of determining which of the three situations shown in fig. 1 one is most likely to encounter in a given area and for a given season. Unfortunately, detailed measurements of water column properties as a function of range are generally not available, and, consequently, we can only guess whether we are dealing with a situation similar to that of figs. 1(a), (b), or (c). This lack of information about the real ocean environment clearly affects the reliability of acoustic propagation predictions. However, lower source frequencies are preferable in order to minimize boundary and volume scattering effects.

If we now turn to the seabed which constitutes the lower boundary of the ocean waveguide, we are again dealing with a strongly inhomogeneous medium, as illustrated in fig. 2. Shown here is a seismic profile of the seabed in a shallow-water area in the Mediterranean. The track is approximately 3 km long and the water depth varies from around 40 m near the shore to 200 m at long ranges with a mean slope of approximately 2.5°. Figure 2 reveals a very complicated bottom structure with the upper layers (< 100 m) running almost parallel to the water/bottom interface, but with strongly inclined deeper layers.

Since sound interaction with the sea floor increases with decreasing frequency, detailed information about the spatial variability of the geoacoustic properties (speed and attenuation of compressional and shear waves plus the material density) is essential for reliable predictions of low-frequency propagation in the ocean waveguide. However, since the geoacoustic parameters are generally not known to the accuracy needed for reliable low-frequency propagation prediction, we essentially end up with an intermediate band of frequencies (from a few tens of hertz to a few hundred hertz in deep water) which clearly favors the predictability of sound propagation in the ocean.

To get a clearer picture of the environmental effects on sound propagation in the sea, let us briefly review the acoustics of the ocean waveguide. Figure 3 is a schematic of some important sound propagation paths: two possible sound-source locations are on the left, and sound is propagating from left to right. The two dashed lines at 0 and 80 km range are sound-speed profiles that indicate two of the innumerable ways in which the sound speed can vary with depth from place to place (or from time to time). Lines A, B, C, and D represent four possible sound paths whose shape is determined by the location of the source and the sound-speed structure over the extent of the propagation.

Path A from the shallow source is surface duct propagation, because the sound-speed profile is such that sound is trapped near the surface of the ocean. Paths B, C, and D are from the deeper source. Ray B, leaving the source at a small angle from the horizontal, tends to propagate in the deep sound channel without interaction with the sea surface or the sea floor. At slightly steeper angles (path C) we have convergence zone propagation, which is a spatially periodic phenomenon of zones of high sound intensity near the sea surface. Here sound interacts with the surface but not with the bottom. Path D is the bottom bounce path, which has a shorter
cycle distance than the convergence zone path. The right-hand side of fig. 3 depicts propagation on the continental shelf (shallow water) where a complicated bottom structure combined with variable sound-speed profiles result in rather complicated propagation conditions not always suited for a ray representation.

Our ability to model acoustic propagation effectively in the ocean is determined by the accuracy with which acoustic loss mechanisms in the ocean environment are handled. Aside from geometrical spreading loss (spherical, cylindrical, etc.) the main loss mechanisms are volume absorption, bottom reflection loss, surface and bottom scattering loss.

Volume absorption in sea water, caused by viscosity and chemical relaxation, increases with increasing frequency. This loss mechanism is the dominant attenuation factor associated with path B in fig. 3, since this path does not interact with the boundaries. Because there is very little volume absorption at low frequencies, deep-sound-channel propagation has been observed to thousands of kilometers.

When sound interacts with the sea floor, the nature of the bottom becomes important. Figure 4 depicts simple bottom-loss curves, with zero loss indicating perfect reflection. For an ideal homogeneous fluid bottom without volume absorption (non-lossy) we still get severe reflection loss above a certain critical angle \( \theta_c \) due to transmission into the bottom. For a real lossy bottom we never get perfect reflection, even though the curves look similar. The steep-angle energy radiated from a source (path D in fig. 3) suffers severe reflection loss and will therefore become highly attenuated after just a few bottom bounces. On the other hand, for shallower propagation angles, many more bounces are possible. A characteristic feature of long-range propagation in shallow water (path E) is that most of the energy propagates close to the horizontal. In reality, the ocean bottom is a layered elastic medium (see fig. 2), which supports both compressional and shear-wave propagation; in this case bottom loss becomes a complicated function of frequency and grazing angle.

A rough sea surface or sea floor causes scattering of the incident sound. The result is a decay of the mean acoustic field in the water column as a function of range (scattering loss), with the scattered energy being lost to the ocean bottom through steep-angle propagation. The scattering loss increases with increasing frequency, and the propagation paths mainly affected are paths A and C (surface scattering loss) and paths D and E (surface and bottom scattering loss).
A consistent mathematical model of sound propagation in the ocean must contain the physics that govern the above-mentioned propagation and loss mechanisms. A sketch of the environmental inputs needed for a simplified but realistic description of the ocean waveguide is given in fig. 5. In this model the ocean consists of a water column of depth $H_0$ limited above by a rough sea surface and below by a rough sea floor. The sound speed $c_0$ in the water column may vary arbitrarily with depth, while density $\rho_0$ and attenuation $\beta_0$ are considered constant. Even though real ocean bottoms exhibit a complicated layering, it has been established that a simple two-layered geoaoustic model generally provides the necessary degrees of freedom to accurately include bottom effects in numerical models for many ocean areas. Hence the bottom may consist of just a sediment layer of thickness $H_1$ and a semi-infinite subbottom. The model should allow for sound speed, density, and attenuation to vary arbitrarily with depth in the sediment layer, while the subbottom can be considered homogeneous. It is desirable that the model can handle shear-wave propagation in both bottom layers. Finally, it must be pointed out that environmental parameters do not vary only with depth but also with range. Hence in real oceans all of the parameters given in fig. 5 may vary significantly along the propagation track.

3. Classification of wave-theory models

In the hierarchy of computer-implemented solutions of the wave equation shown in fig. 6, we have indicated in dashed boxes the classical ray techniques, which shall not be dealt with here, since they are treated in detail in most textbooks on underwater acoustics [1–4]. The ray techniques are computationally efficient, but they only provide approximate solutions for the acoustic wave field. The other techniques outlined in fig. 6 are all wave-theory based, i.e. they fully account for diffraction and other wave effects.

The starting point for all the acoustic models is the wave equation in cylindrical coordinates $(r, z, \theta)$ for a harmonic point source with time dependence \( \exp(-i\omega t) \),

\[
\nabla^2 \psi(r, z) + \left( \frac{\omega}{c(r, z)} \right)^2 \psi(r, z) = -\frac{\delta(r)\delta(z - z_0)}{2\pi r}
\]

(1)

where we have assumed azimuthal symmetry and hence no dependence on the $\theta$-coordinate. Here, $\psi$ is the velocity potential, $\omega$ is the angular frequency of the source placed on the axis of symmetry at depth $z_0$, and $c$ is the sound speed in the medium. This partial differential equation must be solved with the appropriate pressure release and radiation boundary conditions.

Despite the continuous advances in computer technology, a direct numerical solution of (1) for general 2D ocean acoustic problems cannot be done in reasonable time, and, consequently, simplifying assumptions about the environment (1D spatial variability, fluid bottoms only) or approximations to the acoustic wave equation must be introduced to arrive at practical acoustic models.

The most widely used wave-theory models are schematically represented in fig. 6. Depending on the environmental complexity, we can subdivide these models into three categories: (i) solutions for range-independent fluid/solid environments comprising Fourier integral (FFP) and normal mode techniques; (ii) solutions for range-dependent fluid environments comprising coupled mode and parabolic equation techniques; (iii) solutions for general fluid/solid range-dependent environments obtained by direct integration of the wave equation by finite-difference and finite-element techniques. While the computational requirements for this last class of models is prohibitive except for extremely low-frequency short-range problems, the first two categories of solution techniques have gained widespread use over the past 10 years. Below we shall briefly describe the derivation of these solution techniques for time-harmonic problems, starting with range-independent wave theory.
4. Time-harmonic solutions of separable problems

If we assume the ocean waveguide to be horizontally stratified, i.e. the sound speed $c(z)$ varies only with depth, eq. (1) can be solved by separation of variables (1D problem) in an orthogonal coordinate system. This class of problems typically consists of a series of plane-parallel fluid/solid layers terminated above by a vacuum halfspace and below by a solid halfspace, as illustrated in fig. 5.

4.1. Fourier integral

A full-spectrum solution for the acoustic field in a plane-parallel waveguide can be obtained through a decomposition of the field into an infinite set of horizontally propagating cylindrical waves

$$
\psi(r, z) = \int_0^\infty g(k, z) H_0^{(1)}(kr)k \,dk
$$

(2)

where $k$ is the horizontal wavenumber, $g(k, z)$ is the depth-dependent Green’s function and $H_0^{(1)}(kr)$ is the Hankel function of order zero for outgoing waves. By substituting eq. (2) into eq. (1) we see that $g(k, z)$ must satisfy the depth-separated wave equation

$$
d^2g \over dz^2 + \left[\left(\frac{\omega}{c(z)}\right)^2 - k^2\right]g = -\delta(z - z_0).
$$

(3)

To obtain the acoustic field at various depths as a function of range, we approximate the Hankel function in eq. (2) by its asymptotic form and evaluate the integral numerically by means of an FFT. However, before doing this it is necessary to solve eq. (3) with appropriate boundary conditions for a large number of discrete $k$-values covering the entire (but finite) spectrum of horizontally propagating cylindrical waves. For separable problems the Fourier integral technique, also referred to as the fast field program (FFP) technique, constitutes a benchmark solution against which other approximate solutions can be checked.

The first implementations of this solution technique dates back more than 15 years [7,8], but several codes with improved computational efficiency are today available within the underwater acoustics community. Thus a recently developed well-documented FFP code [9,10] has been successfully applied to a variety of low-frequency seismo-acoustic problems, clearly demonstrating the power of the Fourier-integral modeling technique.

4.2. Normal modes

The alternative to a direct numerical integration of eq. (1) is to expand $g(k, z)$ into a complete set of normal modes, $g(k, z) = \sum A_n(k_n u_n(z)$, where the $u_n$’s are solutions of the eigenvalue equation

$$
\frac{d^2u_n}{dz^2} + \left[\left(\frac{\omega}{c(z)}\right)^2 - k_n^2\right]u_n = 0
$$

(4)

which must be solved with the appropriate boundary conditions. The spectrum of eigenvalues generally consists of both a discrete part (trapped modes) and a continuous part (leaky modes). By ignoring the leaky modes, which correspond to sound interacting with the bottom at angles above the critical grazing angle (see fig. 4) and hence are suffering high reflection losses, the field solution can be written as a modal sum

$$
\psi(r, z) = \sum_{n=1}^{M} u_n(z_0)u_n(z) H_0^{(1)}(k_n r).
$$

(5)

Although this solution is complete only for waveguides with perfectly reflecting boundaries, it is a useful approximate solution to a wide class of long-range propagation problems in underwater acoustics.

Normal-mode models have been in use since the early 1970s and by now they clearly constitute the most popular wave-theoretic modeling tool for range-independent propagation situations. Several well-documented codes are available in the community, e.g. [11,12], some of which utilize very efficient modal solvers [13], that can provide up to an order-of-magnitude reduction in calculation time compared to standard solution techniques. This, in connection with the general increase in computational power, has promoted the use of normal-mode techniques in long-range propagation modeling in place of classical ray techniques.

5. Time-harmonic solutions of non-separable problems

If we consider non-separable problems where the sound speed $c(r, z)$ is allowed to vary with both depth and range (2D range-dependent problems), we must distinguish between purely fluid media and mixed fluid/solid media. The first class of problems can be solved by stepwise-coupled normal modes or by the parabolic equation approximation, as discussed in detail below. The class of 2D problems involving mixed fluid/solid media can be solved only by computer intensive finite-difference or finite-element techniques, which shall not be discussed in this paper.
5.1. Coupled modes

A complete two-way solution for wave propagation in fluid media with 2D inhomogeneity can be formulated in terms of stepwise-coupled normal modes [14,15]. As shown in fig. 7, the medium variability in the r-direction has been discretized by subdivision into a number of range segments each with range-invariant properties, but with allowance for arbitrary variation of environmental parameters with depth within each segment. The solution for the acoustic field can be formally written as a sum of local modes with unknown excitation coefficients $A_{j,n}$ and $B_{j,n}$ giving the amplitudes of both outgoing and incoming cylindrical waves

$$
\psi_j(r,z) = \sum_{n=1}^{M} \left[ A_{j,n} H_0^{(1)}(k_{j,n}r) + B_{j,n} H_0^{(2)}(k_{j,n}r) \right] u_n(z,k_{j,n})
$$

where index $j$ refers to the segment number and $n$ to the mode number. The functions $u_n(z,k_{j,n})$ are local modes, which are determined by solving an eigenvalue equation similar to eq. (4) for each range segment. The unknown coefficients $A_{j,n}$ and $B_{j,n}$ are determined by requiring continuity of pressure and horizontal particle velocity across segment boundaries. This involves quite complex expressions with coupling integrals of the form [14]

$$
C_{j,mn} = \int_0^H \frac{1}{\rho(z)} u(z,k_{j,m}) u(z,k_{j+1,n}) dz
$$

(7)

to be evaluated for all modes at all segment boundaries.

The coupled mode technique does require considerable computational power and is therefore not a practical tool for solving general 2D propagation problems. However, since the technique provides complete wave solutions with backscattering included, it is useful as a benchmark for checking approximate 2D solutions.

5.2. Parabolic equation

An approximate solution to 2D propagation problems can be obtained from the parabolic wave equation, which is derived by assuming a field solution of the form $\psi(r,z) = \phi(r,z) \exp(ik_0 r) r^{-1/2}$, where $k_0$ is an average horizontal wavenumber of the propagating wave spectrum. By substituting this expression for $\psi$ into eq. (1) in a source-free region and introducing the paraxial approximation, we arrive at the parabolic wave equation

$$
\frac{\partial^2 \phi}{\partial z^2} + 2ik_0 \frac{\partial \phi}{\partial r} + k_0^2 (n^2 - 1) \phi = 0
$$

(8)

where $n(r,z) = k(r,z)/k_0$ is the refraction index. This partial differential equation is amenable to a marching solution based on either split-step FFT [17,18] or finite-difference techniques [19]. The classical split-step FFT solution takes the form

$$
\phi(r + \Delta r, z) = \exp \left( \frac{ik_0}{2} (n^2 - 1) \Delta r \right) \mathcal{F}^{-1} \left\{ \exp \left( \frac{-i \Delta r}{2k_0 s^2} \right) \mathcal{F} \{ \phi(r, z) \} \right\}
$$

(9)

where $\mathcal{F}$ denotes the Fourier transform from the $z$-domain to the $s$ (vertical wavenumber) domain, and $\mathcal{F}^{-1}$ is the inverse transform.

The parabolic equation technique dates back to the early 1970s [17] and has undergone several refinements and improvements over the years. There are now several parabolic equation (PE) codes available in the community, e.g. [19,20], each solving a particular form of the parabolic wave equation. Today the PE technique is without doubt the preferred wave-theoretic model for solving range-dependent problems. However, it is an approximate technique that neglects backscattering and, more importantly,
assumes energy to propagate within a limited angular spectrum of ±40° with respect to the horizontal. In any event the PE technique has successfully been applied to a multitude of low-frequency ocean acoustic problems with strong range dependence.

6. Pulse solutions by Fourier synthesis

While time-series analysis and modeling has been the standard approach in geophysics for more than a decade in studying low-frequency seismic wave propagation in the earth’s crust, underwater acousticians have traditionally favored spectral analysis techniques which provide information only about the band-averaged energy distribution in space. There are several reasons for choosing this approach in ocean acoustics. Most importantly, the ocean is characterized by high temporal variability, a fact that causes strong (and unpredictable) signal fluctuations for long-range propagation at traditional sonar frequencies (> 1 kHz). At best only the mean signal energy seems to have a predictable behavior at these frequencies. Over the past few years, however, the trend in sonar development has been toward lower frequencies, which should lead to both higher signal stability and predictability. As a consequence the powerful time-series analysis techniques from geophysics might turn out to be a valuable tool also for studying the complex propagation situations encountered in the ocean.

Work on direct simulation of broadband signal dispersion in an ocean waveguide has been under way for a couple of years now. There are fundamentally two different approaches to this modeling problem. The first is to solve the pulse propagation problem directly in the time domain, which requires the development of entirely new propagation codes [21]. Alternatively, one can obtain pulse solutions \( F(r, z, t) \) by Fourier synthesis of harmonic results as [10]

\[
F(r, z, t) = \int_{-\infty}^{\infty} f(\omega)\psi(r, z, \omega)e^{-i\omega t} \, d\omega
\]  

(10)

where \( f(\omega) \) is the source spectrum and \( \psi(r, z, \omega) \) is the spatial transfer function. This approach is attractive since it requires little programming effort. In fact any of the wave-theory codes described in section 5 can be linked up with a pulse post-processor which numerically integrates eq. (10) by an FFT. The spatial transfer functions are generated by repeated execution of the time-harmonic code for a number of discrete frequencies within the frequency band of interest. Clearly, a pulse calculation by Fourier synthesis is computationally slow since typically 100 frequency samples are required to synthesize a pulse result. However, there is no indication so far that direct solution of the problem in the time domain has computational advantages.

Numerical results for pulse propagation modeling using Fourier synthesis are presented in section 7.2.

7. Numerical results

The major difficulty in providing reliable propagation predictions in ocean acoustics is related to the lack of detailed environmental information for both the water column and the seabed. While the spatial variation of sound speed in the ocean itself can be measured in a quasi-synoptic manner by dropping expendable bathythermographs from an airplane, information on the geoaoustic parameters of the seabed are difficult to obtain. There are in practice two different approaches to solving this problems:

- One is direct measurement of wave speeds and attenuations in situ (in bore holes or with buried sensors of some kind) or measurement on core samples. This is a complex process giving ground-truth information at a particular location on the bottom. However, several measurements along the acoustic track will normally be required to determine the lateral variability of the geoaoustic parameters.

- The other is the use of remote sensing techniques, where acoustic data are inverted to determine the acoustic properties of the sea floor. These techniques will generally provide some spatial averaging, particularly if the data used are long-range propagation data instead of local bottom reflection-loss data. The inversion, however, may not provide all the needed information (data could be insensitive to, for instance, the bottom shear properties), in which case an alternative data set would be required.

With the present state of knowledge, we feel that the only feasible way to construct an average geoaoustic bottom in a given part of the ocean is to combine several of the above-mentioned techniques. Thus, we need (a) a few core to obtain ground-truth information from selected sites, (b) seismic profiling for information on the deeper layering, (c) a broadband long-range propagation experiment, and (d) a reliable numerical model for inversion of the acoustic data, e.g. through iterative forward modeling.

Some initial rough information on bottom composition, layering, etc. is needed, and this information can best be obtained from coring and seismic profiling. Then the data/model comparison is used to “fine-tune” bottom parameters until an acceptable agreement is obtained between theory and
experiment. For broadband data with a good depth and range coverage, the "solution" for the ocean bottom should be unique. However, with too many unknown parameters and a limited data set, one may find several parameter combinations that seem to describe equally well the measured acoustic properties of the bottom.

In this section we shall illustrate to what accuracy we can determine the average geoacoustic properties using the modeling procedure described above. Two different data sets are available, both obtained using explosive sources. First we consider a broadband (50–2540 Hz) acoustic data set processed in 1/3-octave bands. The modeling of the band-averaged energy distribution in space was here done as a time-harmonic modeling at the center frequency of each 1/3-octave band. The second data set is associated with propagation of an interface wave along the sea floor and consists of time series recorded on a bottom-mounted geophone. Here a full modeling of the time series was done in combination with dispersion analyses.

7.1. Time-harmonic modeling of acoustic data

We first consider propagation in a quite complex environment in the North Atlantic where the water depth, the sound-speed profile, and the bottom properties vary with range, as illustrated in fig. 8(a). We see that the water depth varies between 115 and 305 m over the length of the track, with a maximum bottom slope of approximately 1°. A series of sound-speed profile measurements in the water column revealed two distinctly different types of profiles along the track: an almost isovelocity profile out to the position of the ocean front (25 km) and then a profile characterized by a higher sound speed in the upper 30 m of the water column. Thus the ocean front is the separation between two different water masses.

Information from coring and seismic profiling indicated that the bottom could be divided into two different types: a homogeneous hard sand bottom on the first 15 km followed by a softer layered bottom consisting of silt overlying sand. Initial estimates of the geoacoustic parameters were obtained from the cores, while the final values (see Table 1) were determined through trial-and-error until an acceptable agreement was obtained between calculated and measured propagation losses over the full frequency band. Since this environment has only weak range dependence (1° bottom slope) the modeling was done with adiabatic mode theory [12]. In the modeling we included both sea surface and sea floor roughness as well as shear wave propagation in the subbottom.

The experimental data with source and receiver both at 50-m depth are shown in fig. 8(b), while the model results are displayed in fig. 8(c). Prop-

Fig. 8. Model/data comparison for propagation loss in coastal water area. (a) Schematic of the environment. (b) Contoured 1/3-octave band data versus range and frequency. (c) Model result.
Table 1
Final values of the geoacoustic parameters.

<table>
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<th>Parameter</th>
<th>Hard bottom</th>
<th>Soft bottom</th>
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<tr>
<td>Compressional speed (m/s)</td>
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</tr>
<tr>
<td>Density (g/cm³)</td>
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<td>Compressional attenuation (dB/λ)</td>
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<tr>
<td>Sub-bottom</td>
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<td>Compressional attenuation (dB/λ)</td>
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<td>Shear attenuation (dB/λ)</td>
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</table>

The existence of an optimum frequency is a general phenomenon associated with ducted propagation in the ocean [22]. It occurs as a result of competing propagation and attenuation mechanisms at low and high frequencies. In the high-frequency regime we have increasing volume and scattering loss with increasing frequency. At lower frequencies the situation is more complicated. With increasing wavelength the efficiency of the duct to confine sound decreases. Hence propagation and attenuation mechanisms outside the duct (in the sea floor) affect the intensity of sound in the duct. In fact, the increased penetration of sound into the lossy ocean bottom with decreasing frequency causes the overall attenuation of sound in the water column to increase with decreasing frequency. Thus we get high attenuation at both high and low frequencies, while intermediate frequencies have the lowest attenuation. In typical shallow-water areas of 100-m depth the optimum frequency of propagation is around a few hundred hertz [22]. The optimum frequency decreases almost linearly with increasing water depth, and it is therefore expected to be of the order of 10 Hz in deep water environments.

In fig. 8(c) we see that the model result has the correct optimum frequency, which increases slightly with range. We also notice an extraordinary agreement on details between the theoretical and experimental results. Thus in propagating over the slope from range 40 to 50 km, there is an abrupt fall-off in sound level (close spacing of contour lines), which is actually geometrical spreading resulting from the increasing depth of the propagation channel. This is followed by better propagation conditions beyond 50 km, where the greater water depth results in less sound interaction with the bottom.

This example demonstrates that even in an extremely complicated propagation situation where all environmental parameters vary with range, a wave-theory model based on adiabatic mode theory gives a quite accurate description of acoustic propagation over a wide range of frequencies. However, the good agreement between theory and experiment in this case was achieved only after adopting an extremely detailed (but simplified) environmental description in which practically every one of the different propagation and loss mechanisms described in section 2 were included. We clearly do not always get the good agreement seen here when trying to model propagation in shallow water. However, we feel that when there is a lack of agreement it is due mainly to insufficient knowledge about the environment (the sea floor, especially) and not due to shortcomings in the wave models in terms of handling the important propagation and loss mechanisms correctly.
7.2. Pulse modeling of interface-wave data

It is well-established that shear rigidity of the ocean bottom affects propagation of waterborne sound through the coupling of acoustic energy into shear waves. This coupling mechanism is of particular importance in low-frequency shallow-water acoustics, where the excitation of shear waves in the bottom often becomes the dominant loss mechanism for waterborne sound [23]. Under these circumstances a realistic physical model of the ocean bottom is a viscoelastic solid described by compressional and shear-wave velocities, the attenuation factors associated with these waves, and the material density.

While the compressional-wave speed and the density of sediments can be determined by direct methods, the shear-wave properties are difficult to measure. This is because of the usually high attenuation of these waves and because it is difficult to generate a wave that consists of predominantly transverse particle motion. The shear speed and attenuation can be indirectly determined, however, through the measured propagation characteristics of the ocean-bottom interface wave, whose existence is intrinsically related to the shear properties of the sediments [24].

A seismic interface wave is a guided wave propagating along the interface between two media with different shear speeds [25]. The wave is generally given different names according to the media properties involved [26]. Hence when propagating on a free surface of a solid it is called a Rayleigh wave, while it is a Scholte wave when propagating along a liquid/solid interface, and a Stoneley wave when associated with a solid/solid boundary. Note that at least one of the media must be a solid for the interface wave to exist. In the case of a water/sediment interface, the pertinent wave type is a Scholte wave with the following characteristics:

- The wave propagates along the sea floor with exponentially decaying amplitude away from the guiding interface (wave is evanescent in both media).
- Particle motion is elliptical in the depth/range plane.
- There is no low-frequency cutoff.
- Propagation speed and attenuation are closely related to the shear properties of the sediment.

To demonstrate how shear speed and shear attenuation profiles can be determined from Scholte-wave experiments in connection with a numerical model, we turn to the experimental records in fig. 9. Shown here are stacked time signals for both the vertical and the horizontal (radial) particle velocities as recorded by a geophone on the ocean bottom [27]. The source was an explosive charge detonated near the sea floor in 20 m of water.

The charge size was increased with range as indicated by the black dots
which the slowest arrival is the Scholte mode ($M_0$) with its energy centered around 2 Hz. The first shear mode ($M_1$) is well excited, with maximum energy around 2.8 Hz, while the second shear mode ($M_2$) is only weakly excited.

The modeling was done with a recently developed numerical model of seismic wave propagation in horizontally stratified media [9,10]. The aim is to construct an environment that leads to computed dispersion characteristics in agreement with the experimental results of fig. 10. This is again done in a trial-and-error fashion, where environmental parameters are changed in a systematic way until acceptable agreement is obtained between theory and experiment. Since we are interested in determining the shear properties for an unconsolidated sediment, we can a priori fix the compressional-wave properties and densities, which are known to have negligible effect on the propagation characteristics of bottom interface waves [29]. The compressional-wave properties and densities used as input to the seismic model are given in fig. 11. Also shown here are the final choices of shear-speed and shear-attenuation profiles for the bottom.

As a confirmation of the validity of this modeling exercise, we have created synthetic seismograms for both the vertical and horizontal (radial) particle velocities (fig. 12). As can be observed, the computed time series are in close agreement with the experimental results in fig. 9, both with respect to signal shape and total dispersion with range. Note that signal amplitudes cannot be compared since experimental and theoretical results are normalized differently.

The computed low-frequency dispersion curves for the model environment are shown in fig. 13. In the upper diagram we have superimposed the theoretical dispersion curves (dashed lines) on the experimental data, and there is clearly excellent agreement between theory and experiment for the modal arrival structure, which in turn means that we have chosen an
appropriate shear-speed profile. Figure 13(b) shows the relative energy distribution in the first three modes as determined from the numerical model.

Again there is good agreement with the experimental results in fig. 13(a), indicating that the choice of shear-attenuation profile is also appropriate.
It should be pointed out that the inferred shear speeds (100 - 320 m/s) and shear-speed gradients (~4 m/s/m) agree well with values given by Hamilton [30] for sand-silt bottoms. Concerning the shear attenuation profiles, reported data are so sparse that no comparison with the literature can be made.

8. Summary and conclusions

We have presented an overview of the most commonly used wave-theory models in ocean acoustics, pointed out their areas of applicability, and demonstrated their ability to accurately describe low-frequency acoustic propagation in complex ocean environments. It has also been shown that the full complexity of real ocean environments must be considered in the numerical models in order to accurately predict the propagation conditions for a broad range of source frequencies. However detailed environmental information, particularly about the seabed, is not generally available, and the more sophisticated the models become, the more detailed environmental information is needed. Therefore, the biggest difficulty associated with propagation predictions in ocean acoustics is not related to the modeling tools themselves, but rather to the lack of environmental information to be used as input to the numerical models. One possible approach to gathering information on geoacoustic properties of the seabed over extended ocean areas is through acoustic remote sensing. Examples of inversion of both acoustic and seismic data using iterative forward modeling procedures have been shown.

References

model (PAREQ) (SACLANT Undersea Research Centre, La Spezia, Italy, 1985).


