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Short Notes

A Systematic Error in Estimating Surface-Wave Group-Velocity Dispersion Curves and a Procedure for Its Correction

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Abstract A systematic error is known to occur in estimating surface-wave group-velocity dispersion when the multiple filter analysis technique is applied. The error comes from the fall-off of the amplitude spectra at long periods. We propose a method for correcting this error. It consists of using a centroid frequency of the filtered spectrum instead of the central frequency of the gaussian window. The method is especially useful when the group-velocity curve is obtained from stacking of individual frequency-time diagrams. We apply this technique to two data sets. The first one consists of nine seismograms of coastal, subduction-zone earthquakes recorded by a broadband station located in Mexico City. This data set has been previously used to estimate an average crustal structure of southern Mexico. The second data set consists of broadband seismograms recorded in India and has been used to determine an average structure of the Indian peninsular region. Our results show that, in the first case, the systematic error is negligible. This is due to the relatively low decay rate of the spectral amplitudes at long periods. However, in the case of the Indian data, the systematic error of the multiple filter analysis cannot be neglected since it changes significantly the measured dispersion curves and leads to errors in the estimated crustal structure. Our tests show that the proposed method successfully corrects a major part of the systematic error.

Introduction

Since its introduction by Dziewonski *et al.* (1969), frequency-time analysis has been widely used in seismology to measure surface-wave group-velocity dispersion. Here, we consider a version of this technique that is called multiple filter analysis (Dziewonski *et al.*, 1969; Herrmann, 1987; Levshin *et al.*, 1989). It consists of the application of a set of gaussian amplitude filters to the input spectrum, followed by calculation of inverse Fourier transforms. The group arrival times are then estimated from the maxima of the time envelopes. It is known that this method leads to a systematic error in the group-velocity measurements (Levshin *et al.*, 1989). This error arises from the variation of the spectral amplitude, which shifts the central frequency of the filtered spectrum. As a consequence, the measured group velocity is assigned to an incorrect frequency. One way to correct this error is to replace the central frequency of the filter by an instantaneous frequency (Levshin *et al.*, 1989). This approach has been successfully applied in the frequency-time analysis of individual records. However, in many cases it is preferable to stack several frequency-time diagrams since this significantly improves the signal-to-noise ratio (e.g., Campillo *et al.*, 1996; Shapiro *et al.*, 1997). The stacking requires automatic estimation of the true frequency of the filtered spectra. In this case, the use of the instantaneous frequency is problematic because of a possible ambiguity of its estimation. This ambiguity arises because of the presence of different modes. We propose the use of the centroid frequency of the filtered spectrum as an approximation to the true central frequency. We tested the method on two real and one synthetic data sets and found that the method successfully corrected a major part of the systematic error

Systematic Error in Frequency–Time Analysis

As mentioned above, multiple filter analysis is often used to compute group velocities of surface waves (e.g., Dziewonski *et al.*, 1969; Herrmann, 1987; Levshin *et al.*, 1989). The analysis of an individual record consists of the following steps: (a) computation of the Fourier transform of the input signal; (b) multiplication of the complex spectrum by a gaussian filter,

$$H(\omega_0, \omega) = e^{-[(\omega-\omega_0)/a\omega_0]^2} K(\omega), \quad (1)$$

where $K(\omega)$ is the input spectrum, ω_0 is the central frequency of the filter, a is the relative bandwidth, and $H(\omega_0, \omega)$ is the filtered spectrum; and (c) computation of the inverse Fourier transform of the filtered complex spectrum, which results in a frequency-time-dependent function $S(\omega_0,t)$. For a single mode the amplitude of this function at a fixed frequency, $A_s(\omega_0,t)$, is, approximately, a gaussian function of time with the maximum at group time $\tau(\omega_0)$. It is more convenient to use the period-group-velocity representation, which is obtained through a simple coordinate transformation,

$$T = 2\pi/\omega_0, \tag{2}$$

$$u = r/t, \tag{3}$$

where T is a period, u is the group velocity at that period, and r is the event-station distance. The dispersion of group time $\tau(T)$ is related to the dispersion of the group velocity $U(T)$ through the relation

$$U(T) = r/\tau(T). \tag{4}$$

The isoline map of the function $A_S(T,U)$ in the period-group-velocity plane gives a convenient graphical representation of the signal. The location of the maximum of amplitude at each period helps to define the group-velocity dispersion curve.

The resolution of the dispersion curve is improved by stacking since it accumulates the information provided by all the available records and provides a mean dispersion curve for the region of interest. A useful technique involves a logarithmic stacking in the period-group-velocity (frequency-time) domain, as described by Campillo *et al.* (1996) and Shapiro *et al.* (1997).

A systematic error in the frequency-time analysis, which results from the variation of the spectral amplitude, is illustrated in Figure 1. Let us suppose that spectral amplitude decreases at low frequencies (as is generally the case for the spectra of surface waves), and let us consider the consequence of multiplying such a spectrum by a gaussian window with a central frequency ω_0 (Fig. 1a). As can be seen from Figure 1b, the maximum amplitude of the filtered spectra will be shifted to a higher frequency, ω_0. Therefore, the measured value of the group velocity will correspond to the frequency ω_0, but it will be attributed to the frequency ω_0. As a consequence, the dispersion curve will be shifted toward lower frequency. In the case of normal dispersion, the measured values of the group velocity will be systematically lower than the correct ones.

One alternative for correcting this systematic error is to replace the frequency ω_0 by the instantaneous frequency Ω (Levshin *et al.,* 1989). This instantaneous frequency is calculated from the derivative of the phase of the inverse Fourier transform of the filtered spectrum, $\Phi_S(\omega_0,t)$, at the point of its maximum amplitude, $\tau(\omega_0)$:

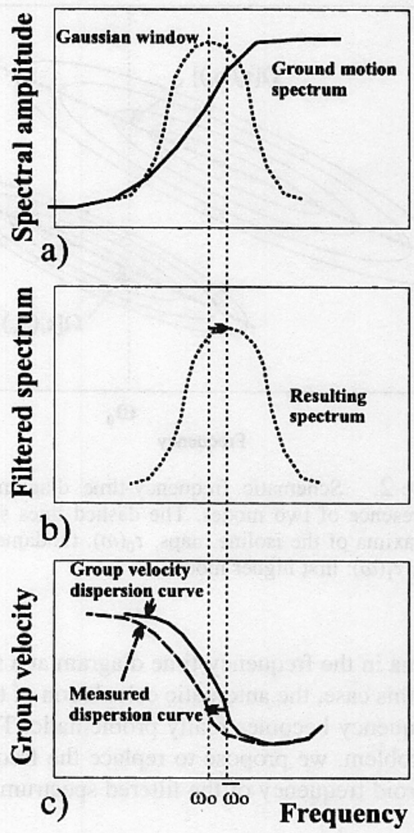

Figure 1. Schematic illustration of systematic error in group velocity measurement resulting from the fall-off of the spectra at low frequencies. (a) Solid line shows amplitudes of the input spectra, and dotted line indicates the gaussian window used in the filter. (b) Amplitudes of the filtered spectra. (c) True (solid line) and measured (dashed line) group velocities.

$$\Omega(\omega_0) = \frac{\partial}{\partial t}\Phi_s'(\omega_0, t)|_{t=\tau(\omega_0)}. \tag{5}$$

During the processing of an individual record, the time, $\tau(\omega_0)$, of the maximum corresponding to the mode that we wish to study can be defined from a visual inspection of the frequency-time amplitude diagram. However, in the case of the stacking of numerous frequency-time diagrams, it is preferable to determine the central frequency of the filtered spectrum automatically. This calculation of the instantaneous frequency using equation (5) can be complicated in the presence of different modes. This situation is illustrated schematically in Figure 2. In this case, the amplitude of the frequency-time diagram has two local maxima at frequency ω_0, and we have, as a consequence, two possible choices of the instantaneous frequency: $\Omega[\tau_0(\omega_0)]$ is calculated at the maximum corresponding to the fundamental mode and $\Omega[\tau_1(\omega_0)]$ is calculated at the maximum corresponding to the first higher mode. Thus, the determination of the instantaneous frequency becomes ambiguous. For a seismogram contaminated by multipathing, we can have more than two

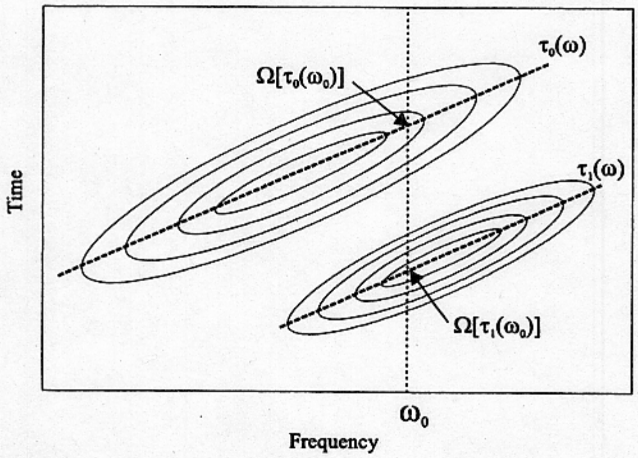

Figure 2. Schematic frequency-time diagram in the presence of two modes. The dashed lines show the maxima of the isoline maps. $\tau_0(\omega)$: fundamental mode, $\tau_1(\omega)$: first higher mode.

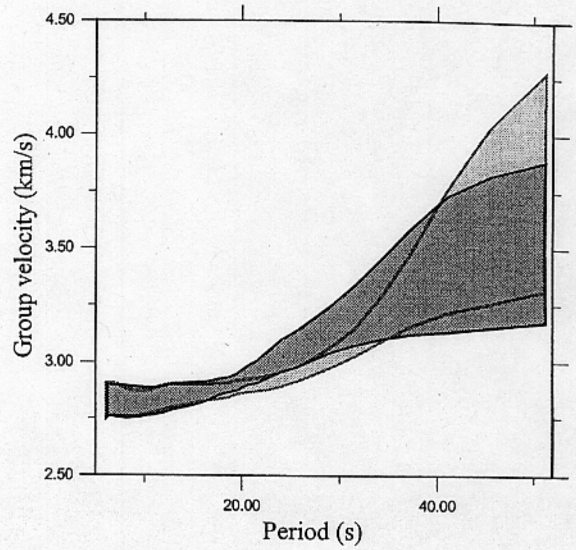

Figure 3. Group-velocity dispersion curves for southern Mexico. The shadowed areas show average and ± 1 standard deviation curves. The light area shows the results of the measurement without the correction of the systematic error. The dark area shows the dispersion curve obtained after the correction of the systematic error.

local maxima in the frequency-time diagram at a single frequency. In this case, the automatic calculation of the instantaneous frequency becomes really problematic. To circumvent this problem, we propose to replace the frequency ω_0 by the centroid frequency of the filtered spectrum, ω_c:

$$\omega_c(\omega_0) = \int \omega |H(\omega_0, \omega)|^2 \, d\omega \tag{6}$$

The calculation of ω_c is simple and unambiguous and can be easily incorporated in the algorithm of stacking in the frequency-time domain.

Application of the Method to Observed Data

Below we show two examples of application of the proposed method on observed data.

Mexico

The data set consists of nine broadband, vertical-component seismograms of earthquakes along the subducton zone of Mexico, recorded at the UNAM campus, located in Mexico City. These seismograms were previously used by Campillo *et al.* (1996) to measure an average dispersion curve of the Rayleigh wave and to invert it for the velocity structure of southern Mexico. In the study of Campillo *et al.* (1996), the correction for the possible systematic error was not applied. In the present study, we corrected the error using the method outlined above. We performed the analysis of displacement seismograms since the spectral fall-off at longer period is less severe than for velocity seismograms. For this reason, we first integrated the velocity seismograms. We then computed the centroid frequencies of the filter using equation (6). In Figure 3, the new dispersion curve is compared with the one given by Campillo *et al.* (1996). At periods between 15 and 35 sec, the corrected group velocities are slightly higher than those previously obtained. However, the difference between the two curves is within the uncertainty of the measurements. Therefore, the errors resulting from the spectral amplitude variation can be neglected.

India

We investigated the influence of the systematic error on the dispersion curve measured in the Indian peninsular shield region (Singh *et al.*, 1999). This data set includes eight seismograms recorded by the newly installed broadband seismographic network of India. We integrated the velocity records to obtain the displacement seismograms and then calculated an average dispersion curve by applying the logarithmic stacking technique in the period-group-velocity domain. We performed calculations ignoring as well including the correction of the central frequency of the filter. Dispersion curves illustrated in Figure 4 clearly show that, in the case of the Indian data, the systematic error results in a significant underestimation of the measured group velocities at periods between 25 and 50 sec. In Figure 5, we present average normalized spectra for the Mexican and Indian seismograms. The decay rate of the spectral amplitude is almost two times greater for the Indian data than for the Mexican data. This explains why the systematic error is large for the Indian shield region but is negligible for southern Mexico.

The Influence of the Systematic Error on the Inverted Velocity Structure

For the Indian shield region, we investigated the effect of the systematic error on the result of the inversion of the

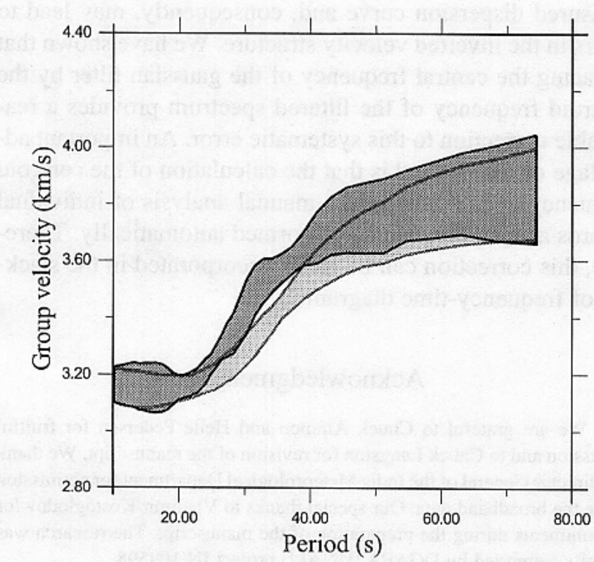

Figure 4. Group velocity dispersion curves for Indian peninsular shield region. The shadowed areas show average and ±1 standard deviation curves. The light area corresponds to the measurements without the correction of the systematic error. The dark area shows the dispersion curve obtained after applying the correction of the systematic error.

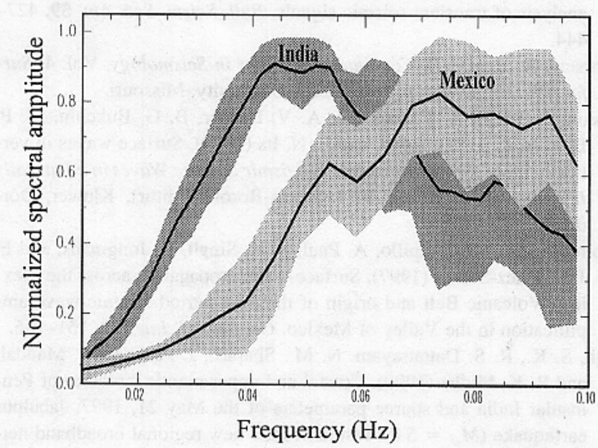

Figure 5. Average normalized spectra of Mexican and Indian seismograms. The solid lines show average values, and the shadowed areas show region of ±1 standard deviation.

measured dispersion curve for the velocity structure. We used the Monte Carlo algorithm in the inversion (Campillo *et al.*, 1996; Shapiro *et al.* 1997). Our initial model included three layers, and we restricted the possible limits of the Moho depth between 38 and 44 km. A detailed description of the selection of parameters of the model are given by Singh *et al.* (1999). As can be seen in Figure 6, the inversion of a noncorrected dispersion curve gives a structure with lower velocities than those obtained from the corrected one. The most striking difference is in the velocity of the upper

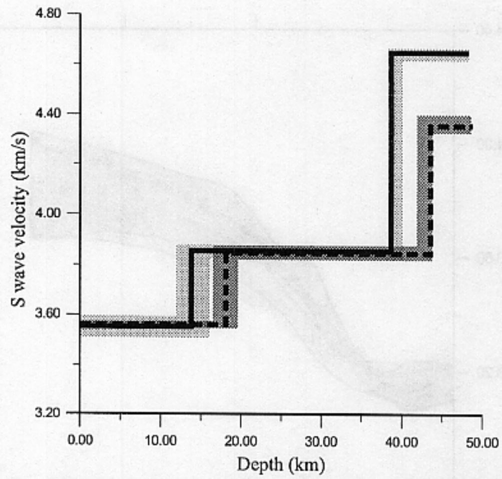

Figure 6. Velocity structure obtained from the inversion of the dispersion curves measured in the Indian peninsular shield region. The dashed line shows the best-fit model found from the inversion of the dispersion curve obtained without the correction of the systematic error. The solid line shows the corresponding model obtained from the dispersion curve corrected for the systematic error. The shadowed areas show the uncertainties of the inversion.

mantle: the *S* wave in the upper mantle is less than 4.4 km/sec if the error is ignored and is 4.65 km/sec if correction is made for the systematic error. We note that S_n velocity reported for the Indian peninsula is 4.61 km/sec (Dube *et al.*, 1973). The structure obtained from the inversion of the corrected dispersion curve is in agreement with the reported S_n velocity.

Synthetic Test

We tested the proposed method on a synthetic data set. We computed complex spectra, $S_i(\omega)$, from the amplitude spectra, $A_i(\omega)$, of the eight vertical-component seismograms of the Indian data set with the help of the following equation:

$$S_i(\omega) = A_i(\omega) e^{-i \int_0^{\omega} R_i/U(\xi) d\xi} \tag{7}$$

where R_i is the epicentral distance of *i*th station, and $U(\omega)$ is the dispersion curve calculated for the average velocity structure of the Indian Peninsula, shown in Figure 6 by continuous straight lines. We note that the corresponding artificial seismograms have the same amplitude spectra as the observed ones from India. We investigated whether the analysis of these artificial seismograms would lead us to the correct dispersion curve. We applied the stacking in the period-group-velocity domain on the corresponding synthetic seismograms. As in the case of the real Indian data, we performed two measurements: one ignoring the correction and one incorporating the correction of the systematic error. In Figure 7, the dispersion curves obtained from the artificial seismograms are compared with the theoretical dispersion

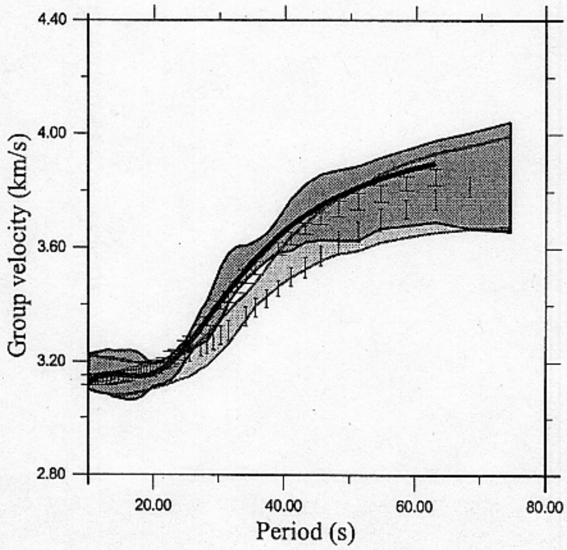

Figure 7. Dispersion curves of a Rayleigh wave measured on the artificial data set. Lower and upper set of vertical bars show the dispersion measured ignoring and correcting the systematic error, respectively. The solid line shows the dispersion curve calculated for the average model of the Indian peninsular shield. Also shown are group-velocity dispersion curves measured in the Indian shield. Lightly shaded and darkly shaded areas show dispersion ignoring and including the correction, respectively (see Fig. 4).

curve and those measured on the real data. If the correction is not applied, then the dispersion curve from the artificial data yields group velocities that are lower than the theoretical ones. On the other hand, the measured dispersion curve, after applying the correction, is nearly coincident with the theoretical one. We conclude that the proposed method satisfactorily corrects the systematic error resulting from the spectral fall-off at low frequencies.

Conclusions

In this note, we have studied the systematic error in the surface-wave group-velocity dispersion curve introduced during frequency-time analysis. The error results from the fall-off of the amplitude spectra at long periods. The analysis of two different data sets shows that the error may be neglected if the decay rate of the spectral amplitude is relatively small. However, when the spectral amplitudes decay rapidly, the systematic error may significantly affect the measured dispersion curve and, consequently, may lead to errors in the inverted velocity structure. We have shown that replacing the central frequency of the gaussian filter by the centroid frequency of the filtered spectrum provides a reasonable correction to this systematic error. An important advantage of the method is that the calculation of the centroid frequency does not require a manual analysis of individual records and, hence, can be performed automatically. Therefore, this correction can be easily incorporated in the stacking of frequency-time diagrams.

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