Collection and three-dimensional modeling of GPS and tilt data at Merapi volcano, Java

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Abstract. We study here the deformations associated with the November 1996 to March 1997 eruption period at Mount Merapi (Central Java), one of the most active volcanoes in Indonesia. This activity period includes a vertical explosion on January 17 and an increase of the lava dome volume by about 3×10^6 m^3. Two Global Positioning System (GPS) campaigns have been carried out on a six-benchmark network at the beginning and at the end of the period. Relative displacements with respect to the reference point show an average subsidence of 6.5 cm. A multicomponent tilt station installed on the southeast flank, 3 km from the summit, recorded a tilt of 11.1 ± 0.7 µrad in the tangential direction and 0.9 ± 0.4 µrad in the radial direction. These data are interpreted using a three-dimensional (3-D) elastic model based on the mixed boundary element method and a near-neighbor Monte Carlo inversion. Interpretation of tilt data requires an accurate mesh for discretizing the 3-D topography. The final result supports a horizontal elliptic magma source located 8.5 ± 0.4 km below the summit and 2 ± 0.4 km to the east of it. In particular, the data cannot be consistent with the location of a magma chamber determined from seismic activity analysis (i.e., 2 km below the summit). The computed depth depends strongly on the source shape and cannot be constrained properly because of the small amount of data. The computed deflation of 11 ± 2×10^6 m^3 is about 3 times larger than the observed increase in the lava dome volume. This difference is attributed to rock avalanches and pyroclastic flows on the flanks of the volcano.

1. Introduction

Some understanding of magma conduits in volcanoes can be achieved from seismic tomography, gravity, or magnetic fields analysis. The study of ground deformations (displacements, tilt, and strain) also contributes to this understanding, but it requires observations before and after eruption periods of interest. Recent synthesis of volcanic geodesy has been conducted by Dvorak and Dzurisin [1997]. One of their main conclusions is that geodesy yields estimates of magma supply rates, the location of sources, and, in some cases, the size and shape of complex magma reservoirs. This has been done only at a few dozen of the world’s 600 active volcanoes.

In this paper, we address Mount Merapi, an andesitic stratovolcano located on Central Java, Indonesia (see Figure 1). Since 1992, Merapi experienced quasi-continuous extrusion of lava at its summit, which forms a dome in a horseshoe-shaped crater. The dome is continuously and partially destroyed by avalanches and pyroclastic flows [Tjetjep and Wittiri, 1996]. During the period November 1996 to March 1997 (about 150 days), the lava dome grew by about 3.2×10^6 m^3, while 126 pyroclastic flows and 16,200 rock avalanches have been recorded (see Figure 4b). On January 17, 1997, at 1034 LT (UT+7), a vertical explosion occurred, forming an eruption column more than 4 km high. The explosion destroyed at least 1.3×10^6 m^3 of the existing dome (A. Ratdomopurbo, personal communication, 1997). The growth rate of the dome corresponds exactly to the long-term average of lava production at Merapi which is equal to 20,000 m^3 d^-1 [Allard et al., 1995].

We present here an analysis of this 5-month eruption period, based on two types of observations: (1) Global Positioning System (GPS) data measured at the beginning and at the end of the period, and (2) continuous tilt variation signals recorded during the period. Looking for large-scale effects, we do not consider summit deformation data but concentrate on observations from the lower flanks of the edifice. In this domain, elasticity applies and a three-dimensional (3-D) model which takes into account both the real topography and the shape of the magma chamber is proposed. The best solution is sought for with a near-neighbor Monte Carlo inversion method.

2. GPS Measurements

The GPS network is based on existing benchmarks installed by the Volcanological Survey of Indonesia and U.S. Geological Survey for Electronic Distance Measurement (EDM) monitoring [Subandrio et al., 1995]. Figure 1 shows the positions of the six chosen points: reference point JRA0 (observation post Jarakah) located on the older Merbabu volcano, BAB0 (observation post Babadan), DEL1 (close to the tilt station Deles), SEL0 (Selokopo Atas), PUSO (Pusunlondon) and LUL0 (Luluk) at the summit. These benchmarks are actually used as a first-order network for a 10-point GPS and microgravity summit network measured every year since 1993 [Jousset et al., 1998; Beauducel, 1998].

The relatively short dimension of the network (8 km for the longest baseline) allows for the use of single-frequency receivers [Botton et al., 1997]. Each campaign included 14 measurement sessions between every two benchmarks, with only two receivers...
These sessions consisted of 2 to 6 hours of simultaneous recording, depending on the baseline length. Because of the number of receivers, the campaign spent about 3 weeks on the field, and sessions were carried out at different days and times that imply decorrelated ionospheric and tropospheric effects (no systematic error). The redundancy factor $f'$, as defined by Botton et al. [1997], is

$$f' = 0.9 \frac{(r - 1)s}{n - 1}, \quad (1)$$

where $r = 2$ is the number of receivers, $s = 14$ is the total number of sessions (baselines), and $n = 6$ is the number of benchmarks. This factor stands for a degree of confidence associated with point position determination. Our value $f' = 2.5$ is sufficient for a standard small network.

Figure 1. Location of Mount Merapi on Java Island, Indonesia, and schematic geologic map. Positions of Global Positioning System (GPS) benchmarks and baselines measured during each campaign are also indicated: Universal Transverse Mercator (UTM-49); JRA0 Jarakah; BAB0 Babadan; DEL1 Deles; SEL0 Selokopo Atas; PUS0 Pusunglondon; LUL0 Luluk. Tilt station Deles is installed at DEL1, on large pyroxene andesitic lava flows, capped by young pyroclastic deposits.

Because altitude differences between points are relatively large (up to 1600 m), a local meteorological model based on field measurements was used for baseline processing, in order to reduce tropospheric refraction effects [Klein and Boedecker, 1989; Gartner et al., 1989]. During each session, atmospheric pressure, dry temperature, and relative humidity were taken at each point every 15 or 30 min. Parameters were reduced to a single elevation according to the tropospheric vertical gradients for pressure (equation (2a)) [Triplett and Roche, 1983] and temperature at tropical areas (equation (2b)) [Saastamoinen, 1972]. We chose to use a constant value for relative humidity (equation (2c)), equal to the ground value, as suggested by Baby et al. [1988]:

$$P = P_0 (1 - 0.0000226h)^{1.225} \quad (2a)$$

$$T = T_0 - 0.00606h \quad (2b)$$

$$H = H_0. \quad (2c)$$

Wet temperature is then computed for all measurements through thermodynamic equations. Its daily variations show a small standard deviation of $2.6^\circ$ (Figure 2). Therefore, we compute an average value for these three parameters (pressure, dry temperature, and relative humidity) for each time session. The software (Sercel GPSWin) determines the baselines by the double-difference method [Dixon, 1991], including meteorological data for tropospheric delay estimations [Hopfield, 1971]. Note that all integer ambiguities were fixed, which is not the case if we use a standard tropospheric model. For long baselines, only 2 or 3 hours of measurement have been selected and kept for the processing, by excluding periods that show large residues in double difference (probably owing to ionospheric effect). Baseline average errors stand for 1 to 2 cm in horizontal and 2 to 5 cm in vertical, which is commonly observed with single-frequency receivers (P. Segall, personal communication, 1998).

Figure 2. Meteorological data during both 1996 and 1997 GPS campaigns at Merapi. They have been reduced with respect to the 3000-m elevation (see text), and they are presented on a single-day scale in local time. Pressure, dry temperature, and relative humidity are field measurements at GPS points. Wet temperature is computed from thermodynamic equations. It shows a relative stability in time (standard deviation of 2.6 $^\circ$C). Horizontal lines represent theoretical values for tropical area.
Table 1. September 1996 Coordinates and a Posteriori Errors for GPS Network Points

<table>
<thead>
<tr>
<th>Point</th>
<th>Name</th>
<th>Local coordinates UTM -49 WGS84, m</th>
<th>Errors a posteriori σ, m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>East</td>
<td>North</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>085</td>
<td>DEL1</td>
<td>440692.1316</td>
<td>9163972.9254</td>
</tr>
<tr>
<td>090</td>
<td>BAB0</td>
<td>434975.8720</td>
<td>9168041.1995</td>
</tr>
<tr>
<td>100</td>
<td>JRA0</td>
<td>436180.2839</td>
<td>9171235.4989</td>
</tr>
<tr>
<td>105</td>
<td>SEL0</td>
<td>439543.9619</td>
<td>9167528.3108</td>
</tr>
<tr>
<td>107</td>
<td>PUS0</td>
<td>439552.3005</td>
<td>9166838.7134</td>
</tr>
<tr>
<td>120</td>
<td>LUL0</td>
<td>438978.0716</td>
<td>9166537.5339</td>
</tr>
</tbody>
</table>

Coordinates and a posteriori errors are determined by least squares adjustment of computed baselines, and are at 68% confidence level. JRA0 is the reference point, and LUL0 is a summit point used only for adjustment.

Table 2. March 1997 Coordinates for GPS Network Points

<table>
<thead>
<tr>
<th>Point</th>
<th>Name</th>
<th>Local coordinates UTM -49 WGS84, m</th>
<th>Errors a posteriori σ, m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>East</td>
<td>North</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>085</td>
<td>DEL1</td>
<td>440692.1582</td>
<td>9163972.9435</td>
</tr>
<tr>
<td>090</td>
<td>BAB0</td>
<td>434975.8794</td>
<td>9168041.2171</td>
</tr>
<tr>
<td>100</td>
<td>JRA0</td>
<td>436180.2839</td>
<td>9171235.4989</td>
</tr>
<tr>
<td>105</td>
<td>SEL0</td>
<td>439543.9658</td>
<td>9167528.3234</td>
</tr>
<tr>
<td>107</td>
<td>PUS0</td>
<td>439552.3021</td>
<td>9166838.7241</td>
</tr>
<tr>
<td>120</td>
<td>LUL0</td>
<td>438978.0607</td>
<td>9166537.5390</td>
</tr>
</tbody>
</table>

Same comments as Table 1.

All computed baselines (representing 42 vector components expressed in the geocentric reference frame) with a priori errors are reduced to 5-point relative positions by a least squares inversion method (3-D geodetic adjustment, J.C. Ruegg and C. Bougault, unpublished data, 1992). Then final positions of points for each period are obtained in local Universal Transverse Mercator (UTM-49) coordinates together with their uncertainties (see Tables 1 and 2). Because of the decorrelated error sources (antenna setting and atmospheric delay), the sound spatial network configuration (some baselines have a horizontal component of 3 km and a vertical component of 1.4 km), and the amount of data used for inversion, the a posteriori errors are relatively small and reflect more the redundancy of measurements than a priori errors.

Accordingly, the relative displacement vectors of the four points BAB0, DEL1, SEL0, and PUS0 are defined for the period September 1996 to March 1997 (see Table 3, Figures 8 and 9). These vectors reveal a significant global vertical downward movement with a mean value equal to –6.5 cm. Displacements of distant stations BAB0 and DEL1 are large enough to conclude that our reference point also moved. Thus these observed displacements are not absolute but relative displacements.

Table 3. Relative Displacements With Respect to JRA0 for GPS Points Between September 1996 and March 1997

<table>
<thead>
<tr>
<th>Point</th>
<th>Name</th>
<th>Relative Displacements, m</th>
<th>Errors σ, m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>East</td>
<td>North</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DEL1</td>
<td>+0.0266</td>
<td>+0.0181</td>
<td>–0.1252</td>
</tr>
<tr>
<td>BAB0</td>
<td>+0.0074</td>
<td>+0.0176</td>
<td>–0.0472</td>
</tr>
<tr>
<td>SEL0</td>
<td>+0.0039</td>
<td>+0.0126</td>
<td>–0.0636</td>
</tr>
<tr>
<td>PUS0</td>
<td>+0.0016</td>
<td>+0.0107</td>
<td>–0.0256</td>
</tr>
</tbody>
</table>

3. High-Precision Tilt Station

3.1. Methodology

The quality of a tilt station is defined by four characteristics, which are, in order of importance: (1) coupling of the instrument with the ground, (2) small or negligible "non-volcanic" deformation (thermal and rainfall) on the site, (3) high sensitivity and high precision of the instrument and (4) continuity and high resolution of numerical data recording. Concerning the coupling characteristics, the aim is to measure tilt variations which are representative of the motion of a large surface on the edifice. However, tiltmeters, except water tubes, measure tilt variations on a surface area which is usually less than 1 m$^2$. In order to extend this surface to a few tens of m$^2$, a solution is to install several tilt components at different locations (see Figure 3). In order to be validated, all components oriented in the same direction must give the same signal. If this is true, we know that no instrumental effects (like drift) have polluted the signals.

The second important quality of a tilt station is linked to the reduction of meteorological effects on the ground, like temperature and rainfall. This problem has been widely studied in the literature, and the best (but expensive) solution is to bury the instruments as deep as possible, as was done, for example, at Sakurajima volcano in a 290-m-deep borehole [Ishihara, 1990]. On the Merapi, Minakami et al. [1969] showed that the temperature at a depth of 1 m is almost free from diurnal variations (within a range ±1°C). Further, the flanks of Merapi are covered on a large area by a compact and massive rock identified as a 5000 years old pyroxene andesitic lava flow, with a thickness that reaches 200 m at some places [Berthommier, 1990]. In a few locations a few meters of young pyroclastic flows cap this rock
(see Figure 1). This soil layer constitutes an isolator as compared with the higher thermoconductive massive rock. On the interface, horizontal temperature gradient is locally almost equal to zero, and thermomechanical effects are strongly reduced by the soil layer. We chose an accessible location 3 km from the summit, on the southeast flank, for installing the tilt station and the DELI GPS benchmark. At this location, the soil layer thickness is equal to about 1 m, and tiltmeters have been installed directly on the basement rock (i.e., at 1 m depth).

### 3.2. Instrumentation

Five tiltmeters have been installed at three different sites distant from each other by about 10 m (three radial components oriented toward the summit in the N30°W direction and two tangential components, 90° anticlockwise, oriented N120°W, see Figure 3). The tiltmeters are horizontal pendulums based on the Zöllner pendulum principle, and they are made of monolithic welded silica, giving a precision of 10⁻⁷ rad [Blum, 1963; Saleh et al., 1991]. Silica has a very small thermal dilatation coefficient (0.54×10⁻⁶ K⁻¹) and relaxation rate (~2×10⁻⁷ yr⁻¹), and it is not influenced by moisture and corrosion. Moreover, this principle provides a means to vary the instrument sensitivity over several orders of magnitude through the adjustment of the oscillation period of the pendulum [Jobert, 1959]. Each tiltmeter has its own period (from 8 to 15 s), defined during installation. This implies slightly different values of sensitivity for each tilt component (see Table 4). Thus instrumental effects, if they exist, can be identified by comparing the response of the different sensors.

#### Table 4. Deles Tilt Station Instrument Characteristics

<table>
<thead>
<tr>
<th>Instruments</th>
<th>Unit</th>
<th>Sensitivity (mV/unit)</th>
<th>Digital Resolution</th>
<th>2-Min Noise</th>
<th>1-Day Noise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tilt tan. CH379</td>
<td>µrad</td>
<td>51.698</td>
<td>1.15×10⁻⁴</td>
<td>0.0121</td>
<td>0.4106</td>
</tr>
<tr>
<td>Tilt tan. CH427</td>
<td>µrad</td>
<td>36.646</td>
<td>2.00×10⁻⁴</td>
<td>0.0117</td>
<td>0.7835</td>
</tr>
<tr>
<td>Tilt rad. CH376</td>
<td>µrad</td>
<td>32.263</td>
<td>2.14×10⁻⁴</td>
<td>0.0136</td>
<td>0.3163</td>
</tr>
<tr>
<td>Tilt rad. CH380</td>
<td>µrad</td>
<td>41.071</td>
<td>5.40×10⁻⁵</td>
<td>0.0099</td>
<td>0.3888</td>
</tr>
<tr>
<td>Tilt rad. CH429</td>
<td>µrad</td>
<td>13.846</td>
<td>1.00×10⁻⁵</td>
<td>0.0302</td>
<td>0.2693</td>
</tr>
<tr>
<td>Rock temp. LIP</td>
<td>°C</td>
<td>100</td>
<td>1.75×10⁻⁵</td>
<td>0.0050</td>
<td>0.0837</td>
</tr>
<tr>
<td>Resistor bridge</td>
<td>V</td>
<td>1000</td>
<td>1.09×10⁻⁵</td>
<td>0.0968</td>
<td>0.4471</td>
</tr>
</tbody>
</table>

Noise is estimated by the standard deviation on signals after high-pass filtering on two important periods: 2 min for sampling period, and 1 day for daily temperature effects.

A temperature sensor has been placed within the rock, and a resistor bridge simulates the response of a stationary tiltmeter. It reflects all forms of electronic noise (acquisition, amplifiers, battery voltage, etc.). Figure 3 shows the location of each sensor. At the three sites, sensors are protected by a steel box covered by a 5-cm-thick isolator. These boxes are then buried under a 1-m cover of natural soil.

For a perfect recording continuity, two data loggers are used (micro Data Acquisition System (µDAS), 20-bit and four-channel continuous integrators) [Van Ruymbeke et al., 1997] instead of radio-transmitter systems. On each sensor, voltages are converted to frequency modulated signals in order to suppress long cable effects. The µDAS directly records frequency of the signal by a simple counter. Thus the recording system constitutes a perfectly linear integrator, with very high-range digital conversion [Beauducel, 1991]. Each data logger has its own power supply system (solar panel and batteries), and there is no possibility of electronic interaction between them. For a 120-s sample (and integration) period, autonomy is more than 2 months before data downloading. Table 4 gives the values of sensitivity, digital resolution, and noise for each instrument. It shows the relatively high precision of tilt measurement and the limited temperature effects (mean standard deviation of 0.43 µrad on daily variations). By comparison, diurnal tilts at Mount St. Helens were 50 to 200 µrad for typical surface installation and a few microradians for an instrument buried 1 m deep in an artificial vault [Dzurisin, 1992].

On Merapi, Deles station was installed in September 1995, and after various technical problems, it has been operational nonstop since July 1996. However, one radial component (CH380) had to be reinstalled in November 1996 and was still drifting during the period considered here, i.e., November 1996 to March 1997.

#### 3.3. Results

Because of the small but obvious correlation of tilts with ground temperature, each signal has been corrected with a nonstationary linear method (described in the Appendix). This improves the global signal-to-noise ratio by about 10%. Figure 4 shows the relative tilt signals of the station in both directions (two radial and two tangential), for the period bounded by the two GPS campaigns. Figure 4 confirms that the two tangential and the two radial sensors record identical long-term signals and are free of any instrumental effects. The complete independence of...
of components provides a means to measure the real tilting of a 30 m × 10 m area at the site and, more importantly, to estimate an error on the tilt measurement. Short-term variations (with periods less than a week) are not consistent within a range of ± 1 μrad. This suggests that site effects on tilts are less than this value. No significant signal is observed at the date of eruption.

The observed tangential (11.1 ± 0.7 μrad) and radial (0.9 ± 0.4 μrad) averaged tilts and standard deviations indicate a global tilting on site of 11.1 ± 0.4 μrad in the N60°E ± 2° direction. The tilt varies continuously in time but not linearly, as is shown by

4. 3-D Elastic Modeling

The first objective of modeling is to locate the deflation (or inflation) source and to determine the associated magma volume variation that fits the data best. Data consist of four relative displacements (4 times three components) and one tilt variation (2 components), i.e., only 14 data. Given the values of observed tilts and displacements and given the anticipated geometry, location, and loading conditions of the source, an elastic solution has been searched for. As is shown by Dvorak and Dzurisin [1997], elastic modeling on active volcanoes for describing surface displacements has been validated in many cases.

4.1. Forward Problem: Topography Effects

4.1.1. Importance of a 3-D model. Mount Merapi, with its 2964-m altitude and its average slopes of 30° (which reach 57° near the summit), exhibits a real three-dimensional topography. Significant errors on source depth and volume variation estimations are made when using a half space model for interpreting surface deformation on prominent volcanoes [McTigue and Stein, 1984; McTigue and Segall, 1988; Cayol and Cornet, 1998a,b]. Moreover, the asymmetry of the northeast flank and the closeness of Mount Merbabu (3142 m) to the north side prohibit any axisymmetrical approximation. Accordingly, the 3-D mixed boundary elements approach (MBEM) [Cayol and Cornet, 1997] has been adopted for the forward problem computation. It takes into account 3-D topography for the ground surface and magma reservoirs with complex geometry. The loading imposed by the magma chamber is modeled by a small pressure variation acting on the chamber surface. This pressure variation induces a change in chamber volume. The relation between surface displacement and changes in chamber volume are independent of the Young modulus of the medium. Since we are looking for magma chamber volume variation and not pressure, we avoid the difficult task of estimating this elastic parameter.

4.1.2. Mesh of the ground surface. A Digital Elevation Model (DEM) of the Merapi region computed from two SPOT images taken in 1987 has been used. The original DEM contains large areas where data are missing because of clouds on eastern portions of Merapi and Merbabu. These have been corrected by considering several topography points, which have been interpolated [Jousset, 1996]. Moreover, detailed analysis of this DEM has revealed, locally, errors in the order of a few hundred meters on the flank of some valleys, because of oblique sunlight when the images were taken. This is particularly noticeable in the vicinity of the tilt station. These errors may result in false computation for tilt direction and amplitude. Consequently, this area has been replaced by a more accurate digitized topography map [U.S. Army, 1964]. Moreover, this detailed map helped to locate the tilt station very precisely (± 10 m), i.e., within an error consistent with the spatial resolution on tilt data. The surface is meshed by a series of imbricated square grids centered on the tilt station location. The point density of the mesh decreases while going away from the center area. Elements are constituted by triangles of the Delaunay type, and the smallest are 20 m large (see Figure 5a).

The validity of this mesh was controlled a posteriori by examining the results obtained with the final source model (see section 4.4 below). Figure 5b shows the relative tilt variations along 500-m-long cross sections oriented both in radial and tangential directions. It is seen that tilting is very dependent on topography but does not exceed 1 μrad on adjacent elements at the Deles tilt station. Our mesh gridding is consistent with this result.

![Figure 4](image_url)
4.1.3. Shallow magma chamber. A first magma chamber model at Merapi has been proposed after the observation of an aseismic domain located between 1 and 3 km below the summit [Ratdomopurbo, 1995; Ratdomopurbo and Poupinet, 1995]. Volume and conduit length inferred from this reservoir geometry are consistent with the magma flux associated with the lava dome extrusion of the 1994 eruption and the gravity changes observed between 1993 and 1994 [Jousset et al., 1998].

We conducted a first calculation on the basis of this model, with a spherical chamber with radius \( r = 850 \) m centered at depth \( z = 1000 \) m (above sea level). Displacements and tilts associated with an arbitrary volume variation \( \Delta V = 1 \times 10^6 \) m\(^3\) are shown in Figures 6 and 7. The results of this first calculation are as follows: (1) displacement amplitudes are almost centered on the summit and unperturbed by topography. In this model, the GPS reference point is far enough from the maximum displacement so that it may be considered as a real static point. (2) Tilt amplitudes are less than 10 µrad, 3 km away from the summit. (3) Topographic effects on tangential displacement (which are very small) and tilt are clearly seen, although they are equal to zero for Mogi’s [1958] model. Maximum tilt is located on the southeast flank 1.5 km away from the summit. It was also found that near the tilt station, topography effects induce differences up to 300% of amplitude in tilt and 180° in direction as compared to the simplified half-space model.

In order to improve the consistency of this model with the data, we adjusted the only parameter of the model (volume variation) since it has a linear effect on displacements. We obtained a very poor misfit value equal to 110 (see Table 5 and equation (4) for the definition of misfit). Accordingly, we decided to look for a new location for the magma source following an inverse problem method.

4.2. Inverse Problem Method

The inversion method consists of the search for some of the model characteristics by relying only on the forward problem formulation and on the observed data. Here the model is reduced to a set of chosen parameters vector \( \mathbf{m} = \{m_1, m_2, \ldots\} \in \mathbf{M} \), where \( \mathbf{M} \) is the “model space”. For a given model \( \mathbf{m} \) the function which measures the degree of misfit between observed data \( \mathbf{d}_{\text{obs}} = \{d_{\text{obs}}^1, d_{\text{obs}}^2, \ldots\} \) with errors \( \sigma_{\text{obs}} = \{\sigma_1^\text{obs}, \sigma_2^\text{obs}, \ldots\} \) and the values predicted with the model \( \mathbf{d}_{\text{cal}} = \{d_{\text{cal}}^1, d_{\text{cal}}^2, \ldots\} \), is called the “misfit function” \( S(\mathbf{m}) \).

If the forward problem is solved by the equation

\[
\mathbf{d}_{\text{cal}} = \mathbf{g}(\mathbf{m}).
\]
then the misfit function for $N$ data with Gaussian experimental uncertainties is given by
\[ S(m) = \sum_{i=1}^{N} \left( \frac{d_i^{\text{calc}} - d_i^{\text{obs}}}{\sigma_i^{\text{obs}}} \right)^2, \]
which presents a minimum for the best parameter set $m$. The 
"likelihood" function $L(m)$ is defined as the a posteriori probability
of a model:
\[ L(m) = k \exp[-S(m)]. \]

This function helps to define the best parameter set $m$
(expected maximum value) and the "quality" of the solution
(stdandard deviation for each parameter $\alpha$).

The relatively long time required for computing the forward
problem solution with the MBEM (about 1 hour on a Sun Sparc
Ultra 1 station, 96 Mbyte RAM) preempts an "exhaustive search"
method for finding the parameter values. The gradient method
cannot be applied here because of the non linearity of the misfit
function. Thus we opted for a slightly modified Monte Carlo
near-neighbor sampling method (Tarantola, 1987). Starting with
a current model $m_{\text{current}}$ with an associated coefficient equal to 1
(called "weight" in this paper), a new "trial model" is chosen
pseudorandomly within a neighborhood of parameters $N \subset M$. If
the condition
\[ S(m_{\text{trial}}) < S(m_{\text{current}}) \]
is satisfied, the trial model becomes the new current model. If
not, the current model is kept, but its weight is increased by 1.
The parameter intervals that characterize the space $N$ are arbi-
trarily chosen. These interval values influence the speed of
convergence but not the final solution. The collection of current
models $m_1, m_2, \ldots$ and their associated misfit values are a repre-
sentative sample of $M$. The process can be stopped by a condition
on the current model weight, for instance, reaching a chosen
value. The last current model (with maximum weight) is the best
one.

Because in an elastic volcanic structure, displacements are pro-
portional to the magma chamber volume variation, the inver-
sion process can be accelerated in the following manner. Instead
of choosing a random value at each trial model for the volume
variation, this parameter is fixed to unity and adjusted a posteriori
with a coefficient that minimizes the misfit function. This
inversion method needs a priori values for the parameters for
initializing the iterative process. In order to fix the first current
model not too far from the final solution, we consider the elastic
infinite half-space analytical solution, which allows a fast inver-
sion (computation time of the forward problem becomes suffi-
ciently low to explore thousands of models).

4.3. First Model: Spherical Source

Surface displacements associated with a point dilatation in an
elastic semiinfinite space have been described by Anderson
[1936]. This solution has been widely used in the literature for
modeling magma chamber as spherical source approximation
[Mogi, 1958], when the mean radius of the chamber is much
smaller than the depth of it ($a \ll d$, where $a$ is the cavity radius
and $d$ is the depth). McTigue [1987] showed that first-order cor-
rrection for a finite spherical magma body varies like $\varepsilon^3$, where
$\varepsilon = ad$, and reaches only 10% for $\varepsilon = 0.5$. This implies that
a point source is a very satisfactory approximation and that the size
of the sphere has not much significance on surface deformations.

Mogi's [1958] model involves four parameters (depth and horizontal
coordinates of source and change in volume). In our case,
we have to take into account relative displacements; that is,
we must determine the four parameters.

A four-parameter model space has been explored: source po-
osition ($X$, $Y$, $Z$) and volume variation $\Delta V$. The best model corre-
sponds to a deflation source of $24 \times 10^6$ m$^3$, located 6 km deep, 4-
km eastward from the summit (see Table 5). Each parameter can
be associated with a standard deviation estimated from the prob-
ability function $L(m)$. The three components of SEL0 and PUS0
and the two components of tilt are correctly reproduced (error
lower than 1 $\sigma$). On the contrary, vertical component of BAB9
and horizontal orientation of DEL1 displacements are not satis-
factory. Considering that this misfit is due to an effect resulting
from the half-space simplification, we used this solution as first
model for the inverse problem with topography.

For this inversion the source is a real sphere with a 1200-m
radius. It corresponds to a volume of $7.2 \times 10^7$ m$^3$ that has been
calculated so as to fit the ratio determined by Blake [1981], i.e.,
about 3 orders of magnitude larger than the erupted magma vol-
ume. Two hundred and sixty-three forward problems have been
computed with a last current model weight equal to 21. The best
model is a deflation source of 11$ \times 10^6$ m$^3$ located 3.7 km below
the summit and 2 km to the east of it. This confirms the results of
Cayol and Cornet [1998a] that for a 30° slope volcano, Mogi's
[1958] model overestimates the volume variation as well as the
depth of the source (referred with respect to the summit eleva-
tion) by as much as 50%. Horizontal location is slightly different
from the half-space solution, and it is closer to the summit
because of the important topographic effects on tilt. However, the
misfit function is still high, and the topography does not explain
the DEL1 horizontal displacement. Surprisingly, the value of the
misfit is worse when taking into account the topography (5.9
instead of 5.1, see Table 5). In order to try to improve the fit to
the data, an ellipsoidal shape has been considered for the source.

4.4. Second Model: Ellipsoidal Source

In order to determine the a priori model required for initializing
the inversion process, we ran a preliminary inversion corre-
sponding to an ellipsoidal source in an elastic infinite half-space.
Because of its simplicity, the solution for a fault in purely open-
ing mode [Okada, 1985] has been chosen for characterizing the
ellipsoidal source. Indeed, surface deformations associated with
a mode 1 fault discontinuity are very similar to those observed with
an ellipsoidal source, except for shallow depth where fault tip
effects become significant. We associate the opening mode with a
volume increase so that a deflation is modeled by a negative
displacement discontinuity. The dislocation is assumed to be
square with a fixed dip angle, which reduces the number of para-

4.4.1 Vertical sheet. From the same 14 data as in the first
model, we explored a six-parameter model space defined by the
position of the center point of the upper edge of the sheet \((X, Y, Z)\), the side length \(L\), the horizontal orientation (strike) \(S\), and the volume variation \(\Delta V\). Only tilt is correctly adjusted with the opening of a vertical 8-km-deep, 3-km sided square sheet (see Table 5). Displacements are very poorly fitted, and we observed in the various models that displacements and tilts cannot be fitted simultaneously by a single model. Since tilt data are better constrained (relative errors smaller than for displacements), the solution fits the tilt data but not the displacements. It is concluded that data are incompatible with a vertical fault source; that is, if the source has a preferential orientation, it is not in the vertical direction.
Table 5. Solutions Found for Different Models of Magma Source

<table>
<thead>
<tr>
<th>Source</th>
<th>X, km</th>
<th>Y, km</th>
<th>Z, km</th>
<th>S, deg</th>
<th>ΔV, 10^6 m^3</th>
<th>S_{min}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere</td>
<td>0.0^b</td>
<td>0.0^b</td>
<td>-1.9</td>
<td>—</td>
<td>-0.47 ± 0.03</td>
<td>110</td>
</tr>
<tr>
<td>Point</td>
<td>4.4 ± 0.1</td>
<td>-0.4 ± 0.2</td>
<td>-9.0 ± 0.1</td>
<td>—</td>
<td>-23.6 ± 1.4</td>
<td>5.1</td>
</tr>
<tr>
<td>Sphere</td>
<td>2.0 ± 0.3</td>
<td>0.0 ± 0.2</td>
<td>-6.6 ± 0.2</td>
<td>—</td>
<td>-11.0 ± 1</td>
<td>5.9</td>
</tr>
<tr>
<td>Vertical fault</td>
<td>2.7 ± 1.1</td>
<td>-1.6 ± 1.4</td>
<td>-10.0 ± 1.4</td>
<td>-48 ± 12</td>
<td>(+101 ± 66)</td>
<td>8.5</td>
</tr>
<tr>
<td>Horizontal fault</td>
<td>2.8 ± 1.0</td>
<td>-0.9 ± 1.4</td>
<td>-11.9 ± 1.6</td>
<td>—</td>
<td>(-24.3 ± 13)</td>
<td>4.9</td>
</tr>
<tr>
<td>Horizontal ellipsoid</td>
<td>2.2 ± 0.4</td>
<td>-0.1 ± 0.4</td>
<td>-8.7 ± 0.4</td>
<td>—</td>
<td>-10.8 ± 2.2</td>
<td>5.3</td>
</tr>
</tbody>
</table>

\((X, Y, Z)\) are east, north and up positions with respect to the summit, \(S\) is the strike (for vertical fault model), and \(\Delta V\) is the volume variation. Each parameter is given with its standard deviation estimated from the a posteriori probability function of the model space. For elastic half-space models \([\text{Mogi}, 1958; \text{Okada}, 1985]\) the virtual horizontal ground surface has been defined at the GPS reference point \(\text{JRA0}\) elevation (1330 m). Fault modeling refers to a mode I dislocation (opening mode only).

\(^a\) MBEM \([\text{Cayol and Cornet}, 1997]\).
\(^b\) Fixed values.
\(^c\) \text{Mogi} [1958].
\(^d\) \text{Okada} [1985].

4.4.2. Horizontal sheet. With the same parameters the best model yields a fault in deflation, 2.3 km wide and 9 km deep, with a better misfit value than any of the previous models (see Table 5). This solution was used as a first-trial model for the inverse problem with topography.

4.4.3. Final solution. For the ellipsoidal source, three solutions with different volumes for the chamber were computed: same volume as that for the spherical source \((a = b = 1731 \text{ m for horizontal axis and } c = 577 \text{ m for vertical axis})\) and then volumes 8 times and 27 times smaller than that of the sphere. Two hundred and three forward problems were computed with a last current model weight of 72. The best model is a deflation source with a decrease in volume equal to \(10.8 ± 2.2 \times 10^6 \text{ m}^3\), located 5.8 ± 0.4 km deep and 2 ± 0.4 km east from the summit. The minimum misfit value equals 5.3. It is higher than that for the half-space model but better than that for the spherical model with topography (see Table 5). There is no significant difference (source position and volume variation) between the solutions obtained with the three chamber volumes. It is concluded that this modeling cannot resolve the initial source volume. Figures 8 and 9 show the computed displacements and tilt vectors as compared to the data. As for previous models, vertical displacement of BAB0, horizontal direction, and vertical displacement of DEL1 are not reproduced satisfactorily. Figure 10 shows the model space samples for the sphere and for the ellipsoid sources, as a function of volume variation versus depth. It reveals the linear dependency between these two parameters (the shallower the depth, the lower the volume variation) and the independency of the volume variation with respect to source shape and size.

Figure 9. South-north vertical projection of GPS displacements and final solution displacements (same comment as Figure 8).

Figure 10. Model spaces for ellipsoidal and spherical sources, as a function of volume variation versus depth. Dot size stands for probability of each model. A positive correlation appears for both models. The source geometry affects only the depth not the volume variation.

5. Discussion

5.1. Methodology

5.1.1. Tilt modeling. All the tested models adjust exactly the two tilt data, because of the relatively small uncertainty associated with their measurement as compared to that on displacements. These tilt data have an important part on the inversion because they fix the horizontal position of the source; thus they eliminate a set of models which would have been selected had only displacement data been considered. However, we know that tilting is dependent of local surface topography. This entails three conditions if tilts are to be introduced efficiently in modeling: (1)
Tilt data have to be correctly measured and validated in order to represent the real tilting of the sites with large enough area. (2) It is essential to determine the realistic error associated with the measurement, because the inverse problem solution is highly constrained by data uncertainties. (3) Three-dimensional topography has to be accurately digitized and meshed in order to be used in the model.

5.1.2. Model Validity. We were not able to fit our complete data set but only 11 out of the 14 data for a four-parameter model. Nevertheless, this result provides a significant appreciation of the source position and its volume variation. The Monte Carlo inversion method which has been used allows us to determine probabilistic errors (one standard deviation) for each parameter estimation. These errors are equal to few hundred meters for source position and 2.2\times10^6 m^3 for volume variation. These errors stand only for the final solution, i.e., for a fixed source size. We showed that the source depth is dependent with the chamber shape and that variation volume is independent with the chamber size. With the presently limited database it has not been possible to constrain the size of the chamber, but we clearly showed that a vertical shape is improbable. This suggests that below the Merapi at few kilometers depth, the vertical stress component is the minimum principal stress component, a feature consistent with the regional tectonics.

5.2. Volcanological Aspects

The position of the source which has been found is not incompatible with the existence of a shallow magma chamber as proposed by Radomopurbo [1995]. Indeed, the periods of study are not the same, and it may be supposed that the two magma reservoirs exist. Our data reveal that the deeper chamber had a prominent activity during this eruption period and that if it exists, the upper chamber did not sustain any variation in pressure.

The volume variation found by our model (−10.8 ± 2.2\times10^6 m^3) can be compared with the estimated volume of produced lava at the summit. If we suppose that all the deflation volume has been extruded at the summit, only 30% (3.2\times10^6 m^3) of it corresponds to the dome formation. The remainder (7.6\times10^6 m^3) cannot have been ejected instantaneously from the deep reservoir during the explosion because the continuous tilt records show that the deflation of the source was regular (see inset in Figure 4) during the 5-month period. Also, the magma cannot have accumulated in a temporary shallow location during the period preceding the explosion. Indeed, a volume variation of this magnitude would have induced a tilt amplitude equal to about 80 μrad at the Deles station (see section 4.1). Given the tilt signals, the volume variation in the deep magma chamber must correspond exactly to the volume of lava produced at the summit. This difference between calculated and observed volumes (a factor 3) leads to two consequences:

1. On the basis of our model and because the model uncertainty has been well determined, we conclude that the volume of extruded lava has not been estimated correctly. The missing 7.6 ± 2.2\times10^6 m^3 corresponds very likely to the volume of continuous rock avalanches, which occurred during this period. A simple calculation shows that the missing volume can be included in a quadrangular prism 2 km long, 2 km wide at the bottom, 300 m wide at its top (crater) and 5.3 ± 1.5 m thick near the crater, with no thickness at the bottom.

2. The explosion did not involve large magma volumes and was only superficial.

6. Conclusion

Deformation measurements obtained for the period November 1996 to March 1997, provide some constraints on the magma chamber at the Merapi volcano. Taking into account a 3-D topography and including both displacements and tilt observations in an inversion process modifies the conclusions of the classical half-space modeling approach and, hopefully, yields results closer to reality.

We determined that for the period of concern and despite a strong explosion, the deformations were almost continuous in time and are due to a deflation of a deep magma source located some 6 km below sea level. The corresponding magma production can be related only partly to the surface dome growth and suggests that the volume of rock avalanches is equal to about 3 times the volume of the dome growth as detected at the crater level.

This type of study has been possible because (1) a noninterrupted data set for the complete duration of the selected eruption period was available, (2) realistic uncertainties have been determined for each type of data, and (3) a digital elevation model with a spatial precision consistent with the data was available. This limited data set helps only to improve slightly the comprehension of Merapi behavior. However, this methodology should prove efficient for determining precisely the shape and the depth of the magma chamber when more data are available.

Appendix: Temperature Correction of Tilt Signals

Sensitive surface deformation measurements are challenged with a major problem: even if the instrument is exempt from temperature effects, it measures real ground deformation due to meteorological parameters like temperature and rainfall. Attempts to model these effects have shown that the relation-ship between temperature and tilt is usually linear to the first order but variable in time and different for each frequency [Berger, 1975; Goulty et al., 1979; Mortensen and Hopkins, 1987; Desroches, 1990; Beauducel, 1992]. With rainfall the relation-ship is strongly non-linear if the water table level is at shallow depth [Wolfe et al., 1981; Evans and Wyatt, 1984]. On Merapi the water table is about 1000 m below the Deles tilt station (H. Shibano et al., unpublished data, 1994); thus the ground is almost always unsaturated, even a few hours after a hard rainfall.

If S_{obs}(t) is the observed signal (measured tilt) and N(t) is the disturbing signal (measured temperature), then the problem of identifying tilt associated with sources other than temperature can be expressed by

\[ S_{obs}(t) = S(t) + G[N(t - τ)], \]

where S(t) is the tilt free of temperature effects and G is an unknown function.
This problem comes down to estimating \( G \) from \( S_{\text{obs}}(t) \) and \( N(t) \). This cannot be resolved without some a priori information on unknown parameters of equation (7), and the danger is to include characteristics of \( S(t) \) into the function \( G \). Classical autoregressive methods (like the Wiener filter) suppose that \( S(t) \) is a random or a “white” signal [Kofman et al., 1982], but we know that this is not the case with tilt on volcanoes.

A simple method is proposed here for removing most temperature effects from a tilt signal. This method does not imply strong hypothesis on \( S(t) \) but relies strongly on the hypothesis that \( G \) is a linear function of \( N \). Furthermore, every step of the signal processing is performed in the time domain, in order to avoid introducing numerical noise by the time-frequency domain transform. For instance, high-pass filtering with cut-off period \( p \) is based on zero-phase moving average (ZMA) [Oppenheim and Schafer, 1989] defined as

\[
F_p[s(t)] = s(t) - \text{ZMA}_p[s(t)].
\]  

(8)

This process is iteratively repeated with different values for \( p \). For each step, \( p \) decreases by a factor 2. It is realistic to limit the maximum period to the third of the total time interval. Figure A1 presents an example of original and corrected tilt signal, temperature, and the set of correlation functions \( C_p(t) \) for \( p = \{64, 32, 16, 8, 4, 2\} \) days, for a total interval of 142 days. It is obvious that for each period the correlation is nonstationary. Global noise attenuation is about 13% and reaches 63% for daily variations.

Some residues clearly remain in the corrected signal (especially for the 1-week period), but the correlation with temperature is not linear and strongly phase-shifted. Decorrelation of these residues cannot be done without an a priori on the second-order characteristics of function \( G \).

In conclusion, the method is not optimal for a global noise reduction, but it respects short-period events like steps in tilt signal. The process was applied to another period in May 1997, which is much more quiet (no volcanic effects); the residue shows clearly diurnal and semidiurnal tidal waves, consistent in amplitude and phase with theoretical tilt tides [Beauducel, 1998].

This method is presently used with success at Montagne Pelée observatory [Voide, 1997].

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References
Allard, P., D. Carbonnelle, D. Dajlevic, N. Metrich, and J.C. Sabroux, The volatile source and magma degassing budget of Merapi volcano: Evidence from high-temperature gas emissions and crystal melt...


Van Ruymbeke, M., F. Beauducel, and A. Somerhausen, The Environmental Data Acquisition System (EDAS) developed at the Royal Observatory of Belgium, Cahiers du CEGS Luxembourg, 14, 163-174, 1997.
